Universität Erlangen-Nürnberg
Naturwissenschaftliche Fakultät I
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Integer and Nonlinear Optimization
Exercise 7

Problem 1
Let $S = \{\underline{x} \in \mathbb{Z}^2 : 4x_1 + x_2 \leq 28, x_1 + 4x_2 \leq 27, x_1 - x_2 \leq 1, x_1, x_2 \geq 0\}$. Determine the facets of $\text{conv}(S)$ graphically. Then derive each of the facets of $\text{conv}(S)$ as a Chvátal-Gomory inequality.

Problem 2
Solve the following problem by Gomory's cutting plane method.

$$\begin{array}{ll} \max & 7x_1 + 2x_2 \\ \text{s.t.} & \begin{pmatrix} -1 & 2 & 1 & 0 & 0 \\ 5 & 1 & 0 & 1 & 0 \\ -2 & -2 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 4 \\ 20 \\ -7 \end{pmatrix} \\ & \underline{x} \in \mathbb{Z}^5_+ \end{array}$$

Hint:
An optimal basic solution for the corresponding LP is given by $x_1 = \frac{36}{11}, x_2 = \frac{40}{11}, x_5 = \frac{75}{11}$ with objective value $\frac{332}{11}$.

Problem 3
Let $S = \{\underline{x} \in \mathbb{Z}^3_+ : 19x_1 + 28x_2 - 184x_3 = 8\}$. Derive the valid inequality $x_1 + x_2 + 5x_3 \geq 8$ using modular arithmetic.

Problem 4
Determine a valid disjunctive inequality for $S = \{\underline{x} \in \mathbb{Z}^2_+ : x_1 - \frac{5}{4}x_2 \leq -\frac{1}{3}, 3x_1 + \frac{13}{4}x_2 \leq 16\}$ and draw the resulting feasible region.