Universität Erlangen-Nürnberg
Naturwissenschaftliche Fakultät I
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Integer and Nonlinear Optimization
Exercise 3

Problem 1
Prove Lemma 2.7. of the lecture (totally unimodular matrices).

Problem 2
Prove Lemma 2.8 and Lemma 2.9 of the lecture (sufficient condition for total unimodularity and total unimodularity of digraphs and bipartite graphs).

Problem 3
Check whether the following matrices are TU.

$$A_1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}, \qquad A_2 = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{pmatrix}.$$

Problem 4
Show that the following system is not TDI, although it has integer optimal solutions.

$$\min \quad x_1 + x_2 + x_3 + x_4 + x_5 + x_6$$

$$\begin{array}{rrcl} \text{s.t.} & x_1 + x_2 + x_3 & \geq & 1 \\ & x_1 + x_4 + x_5 & \geq & 1 \\ & x_2 + x_4 + x_6 & \geq & 1 \\ & x_3 + x_5 + x_6 & \geq & 1 \end{array}$$

(Hint: Find the optimal solution of the dual.)
Is the constraint matrix totally unimodular?