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Naturwissenschaftliche Fakultät I
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Integer and Nonlinear Optimization
Exercise 12

Problem 1
Consider the problem

$$\min\ x_1^2 - x_2^2$$
$$\text{s.t.} - (x_1 - 2)^2 - x_2^2 + 4 \leq 0.$$

Write down the KKT conditions for this problem, find all solutions satisfying the conditions and find all local minima.

Problem 2
Consider the LP

$$\begin{array}{rcl} \min\ 2x_1 + 3x_2 & & \\ x_1 + x_2 & \leq & 8 \\ -x_1 + 2x_2 & \leq & 4 \\ x_1,\, x_2 & \geq & 0 \end{array}$$

Write down the KKT conditions. Check each extreme point of the feasible set and determine whether or not the conditions are true. Use this to find an optimal solution. Illustrate the problem and the KKT conditions at the extreme points.

Problem 3
Prove Theorem 10.20 of the lecture.
Hint: Use the convexity conditions of Theorem 8.11 and the fact that h_i is affine if and only if both h_i and $-h_i$ are convex. Note that convexity is only needed in one direction.

Problem 4
Let $S \subseteq \mathbb{R}^n$ and $S \neq \emptyset$. The cone of tangents of S at $\overline{x}$ is

$$T_{\bar{x}}(S) := \{d \in \mathbb{R}^n : d = \lim_{k\to\infty} \lambda_k(x_k - \bar{x}) \text{ where } \lambda_k > 0, x_k \in S \text{ and } x_k \to \bar{x}\}$$

(a) Show that if $\bar{x}$ is a local minimum then $F_0 \cap T_{\bar{x}}(S) = \emptyset$ where F_0 is defined as in Theorem 10.8 and f is differentiable at $\bar{x}$.

(b) Let $S = \{x : g_i(x) \leq 0,\ i = 1, \ldots, m\}$ and suppose that f and g_i are differentiable at $\bar{x}$. Show that if $T_{\bar{x}}(S) = G' = \{d : \nabla g_i(\bar{x})^T d \leq 0\, \forall i \in I\}$ and if $\bar{x}$ is a local optimum then there exist $u_i \geq 0$, $i \in I$ such that

$$\nabla f(\bar{x}) + \sum_{i\in I} u_i \nabla g_i(\bar{x}) = 0.$$

(Hint: Use Part a) and Farkas' Lemma).