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A Location Problem in Halle (Saale)

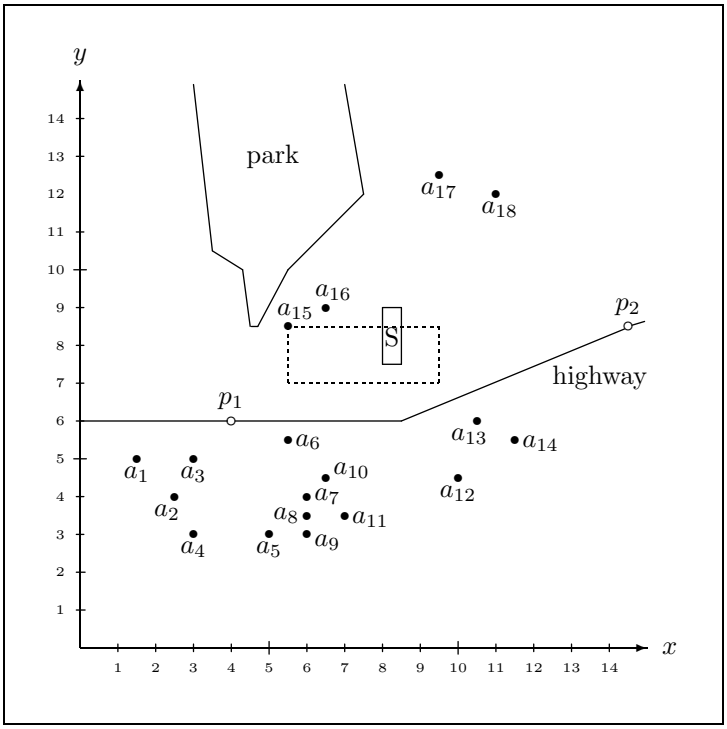

Figure 1: Apartment buildings and geographic information

Problem Formulation

In a residential area of the city of Halle a new playground is to be located. The considered neighborhood includes a total of 18 apartment blocks. Each of these apartment blocks is identified by its coordinates with respect to a map of the area. The approximate number of children living in each of the respective units is provided by the city council. A rough map of the relevant part of Halle is given in Figure 2. Table 1 contains the coordinates of the existing apartment blocks together with the approximate number of children living in each of the buildings.

Coordinates of apartment blocks	Number of children
$(1.5, 5)^T$	58
$(2.5, 4)^T$	38
$(3, 5)^T$	58
$(3, 3)^T$	19
$(5, 3)^T$	115
$(5.5, 5.5)^T$	115
$(5.5, 8.5)^T$	58
$(6, 4)^T$	24
$(6, 3.5)^T$	24
$(6, 3)^T$	48
$(6.5, 4.5)^T$	19
$(6.5, 9)^T$	58
$(7, 3.5)^T$	38
$(9.5, 12.5)^T$	58
$(10, 4.5)^T$	77
$(10.5, 6)^T$	115
$(11, 12)^T$	77
$(11.5, 5.5)^T$	19

Table 1: The coordinates of the entrances to the 18 buildings and the approximate number of children living in each building.

To estimate walking distances in the considered part of Halle, the Manhattan metric l_1 appears to be appropriate since most of the streets in this neighborhood follow a rectilinear pattern.

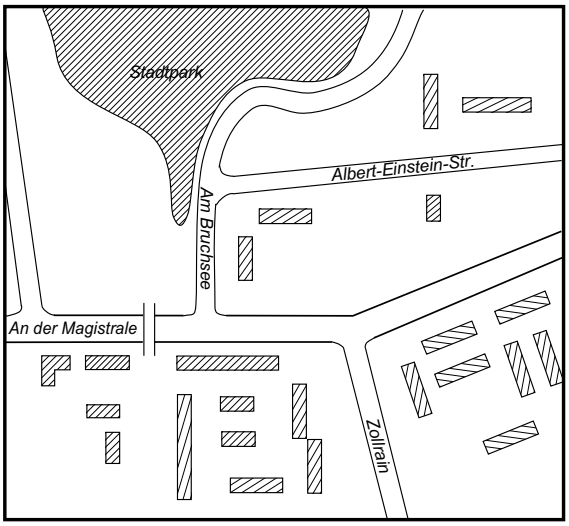

Figure 2: The considered part of the city of Halle.

Additionally, a regional constraint imposed by a highway intersecting the considered neighborhood deserves special consideration. Only two bridges are available that allow a safe crossing of this highway for the children on their way to the playground. To account for the resulting detour on the way to the playground for some of the children, the highway is modeled by an almost linear barrier with two passage points at the location of the bridges.

The resulting model is illustrated in Figure 1, where the notation a_m is used to refer to the apartment buildings (the existing facilities). The highway is interpreted as a piecewise linear barrier with the two passage points $p_1 = (4.0, 6.0)^T$ and $p_2 = (14.5, 8.5)^T$.

There are at least two reasonable suggestions for an objective function:

- Minimize the total walking time for the children to the playground, i.e.,

$$\min \quad f(x) = \sum_{m \in \mathcal{M}} w_m d(x, a_m)$$

- Minimize the maximum over all (weighted?) walking times for the children to the playground, i.e.,

$$\min \quad g(x) = \max_{m \in \mathcal{M}} w_m d(x, a_m)$$

Discrete Model

Five candidate locations representing areas that already belong to the city or that may be purchased at a reasonable cost can be considered for the playground. The candidate locations are represented by the coordinates $(4,3)^T$, $(4,8)^T$, $(6,7)^T$, $(8,4)^T$ and $(12,4)^T$. The city council considers also the construction of two or three playgrounds in case this is indicated by the mathematical analysis.

Continuous Model

A park located in the area must be viewed as a forbidden region for the location of the playground (see Figure 1). The dashed rectangle represents an area that is particularly preferred by the city council for the construction of the playground. A local train station is partially contained in this region, indicated by the small rectangle identified by the symbol S.