Universität Erlangen-Nürnberg
Naturwissenschaftliche Fakultät I
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28.05.2003

Linear and Network Optimization
Exercise 8

Please return your solutions by Tuesday, June 3rd, 10:00 a.m., in the mailbox No. 5.

Problem 1 (7 points)
Formulate the simplex method for LPs with bounded variables as an algorithm, including the update of the reduced tableau.

Problem 2 (7 points)
Apply your algorithm to the LP $\min\{\underline{c}\,\underline{x} \,:\, A\underline{x} = \underline{b},\ \underline{l} \le \underline{x} \le \underline{u}\}$ with

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & 0 & 1 \end{pmatrix}, \quad \underline{b} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

$$\underline{c} = (1,1,2,3,1), \qquad \underline{u} = (3,3,4,2,1)^T, \qquad \underline{l} = (0,0,0,0,0)^T.$$

For the starting solution you can use

$$B = \{1,3,5\}, \quad L = \{2\}, \quad U = \{4\}.$$

Problem 3 (6 points)
Consider the Klee-Minty example of an LP where the simplex method goes through all of the 2^n basic feasible solutions in the worst case (with $0 < \varepsilon < \frac{1}{2}$):

$$\begin{array}{rrcll} \max & x_n & & & \\ \text{s.t.}\quad 0 & \le\ x_1 & \le & 1 & \\ \varepsilon x_{j-1} & \le\ x_j & \le & 1 - \varepsilon x_{j-1} & \forall j = 2,\dots,n \\ & x_j & \ge & 0 & \forall j = 1,\dots,n \end{array}$$

(a) Show that the Klee-Minty problem is equivalent to the following problem, where $\theta := \frac{1}{\varepsilon}$:

$$\begin{array}{lrcll} \max & \sum_{j=1}^{n} y_j & & & \\ \text{s.t.} & y_1 & \le & 1 & \\ & y_j + 2\sum_{k=1}^{j-1} y_k & \le & \theta^{j-1} & \forall j = 2,\dots,n \\ & y_j & \ge & 0 & \forall j = 1,\dots,n \end{array}$$

Hint: Use the transformation $y_1 := x_1$ and $y_j := \frac{(x_j - \varepsilon x_{j-1})}{\varepsilon^{j-1}}$ for $j = 2,\dots,n$.

(b) Graph the two problems for $n = 2$ and $\varepsilon = \frac{1}{3}$.

(c) Apply the simplex method to the reformulation of subproblem (a) and illustrate the path of the algorithm through the basic feasible solutions in the graphs of subproblem (b).