Universität Erlangen-Nürnberg
Naturwissenschaftliche Fakultät I
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Linear and Network Optimization
Exercise 6

Please return your solutions by Tuesday, May 20th, 10:00 a.m., in the mailbox No. 5.

Problem 1: Programming Exercise (7 points)
Based on your implementation of the simplex algorithm in Exercise 5.3, implement the 2-phase method (Algorithm 2.24) for LP's in standard form $\min\{\underline{c}\,\underline{x} \,:\, A\underline{x} = \underline{b},\ \underline{x} \geq \underline{0}\}$ in Matlab. Use Bland's rule (Theorem 2.22) for the selection of the pivot element to avoid cycling. (The programs should be submitted on a floppy disk.)

Problem 2 (3 points)
Apply your algorithm to solve the following problem:
A farmer is raising pigs for the market and he wishes to determine the quantities of the available types of feed that should be given to each pig to meet certain nutritional requirements at a minimum cost. The number of units of each type of basic nutritional ingredient contained within a kilogram of each feed type is given in the following table, along with the daily nutritional requirement and feed costs:

nutritional ingredient	kilogram of corn	kilogram of tankage	kilogram of alfalfa	minimum daily requirement
carbohydrates	90	20	40	200
protein	30	80	60	180
vitamins	10	20	60	150
cost (c)	21	18	15	

Problem 3 (3 points)
Find the dual of the following LPs:

(a)
$$\begin{array}{lrcl} \min & 2x_1 - 7x_2 - 3x_3 & & \\ \text{s.t.} & x_1 + 2x_2 + x_3 & \leq & 5 \\ & 2x_1 + x_3 & \leq & 10 \\ & x_1, x_2, x_3 & \geq & 0 \end{array}$$

(b)
$$\begin{array}{lrcl} \max & -x_1 + x_2 & & \\ \text{s.t.} & x_1 + 2x_2 & \geq & 4 \\ & x_2 & \leq & 4 \\ & 2x_2 - 3x_1 & \geq & 0 \\ & x_2 & \geq & 0 \end{array}$$

(c)
$$\begin{array}{lrcl} \min & \underline{c}\,\underline{x} & & \\ \text{s.t.} & A_1\underline{x} & \leq & \underline{b}_1 \\ & A_2\underline{x} & = & \underline{b}_2 \\ & \underline{x} & \leq & \underline{u} \\ & \underline{x} & \geq & \underline{l} \end{array}$$

Problem 4 (2 points)
Give an example for case (iii) of Theorem 3.5 (b) of the lecture. (Theorem 3.5 (b) (iii): Both (P) and (D) are infeasible.)

Problem 5 (5 points)
Lemma of Farkas: For any $m \times n$ matrix A and any vector $\underline{b} \in \mathbb{R}^m$ either (1) or (2) has a solution but not both.

$$(1)\quad \begin{array}{rcl} A\underline{x} & \leq & \underline{b} \\ \underline{x} & \geq & \underline{0} \end{array} \qquad\qquad (2)\quad \begin{array}{rcl} A^T\underline{y} & \geq & \underline{0} \\ \underline{b}^T\underline{y} & < & \underline{0} \\ \underline{y} & \geq & \underline{0} \end{array}$$

(a) Give a geometric interpretation for Farkas' lemma for $n = m = 2$.

(b) Prove Farkas' lemma (in the general case, i.e., $n, m \in \mathbb{N}$).
(Hint: Consider the LP $\max\{\underline{0} \cdot \underline{x} \,:\, A\underline{x} \leq \underline{b},\ \underline{x} \geq \underline{0}\}$ and its dual.)