Universität Erlangen-Nürnberg
Naturwissenschaftliche Fakultät I
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Linear and Network Optimization
Exercise 5

Please return your solutions by Tuesday, May,13th, 10:00 a.m., in the mailbox No. 5.

Problem 1 (5 points)
Consider an LP of the form $\min\{\underline{c}\,\underline{x} : A\underline{x} = \underline{b}, \quad \underline{x} \geq \underline{0}\}$ to be solved with the simplex method. Let $\underline{x}$ be an arbitrary feasible solution.
Formulate an algorithm for finding a basic feasible solution starting from $\underline{x}$. Apply your algorithm to the following problem:

$$A = \begin{pmatrix} 1 & 1 & 3 & 4 & 0 & 0 & 0 \\ 0 & -1 & -1 & -2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 2 & 0 & 1 & 0 \\ 1 & 0 & 2 & 2 & 0 & 0 & -1 \end{pmatrix}, \quad \underline{b} = \begin{pmatrix} 28 \\ -13 \\ 13 \\ 15 \end{pmatrix}, \quad \underline{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Problem 2 (5 points)
Consider the simplex tableau $T(B)$. Show that whenever a pivot operation is performed to move from a basis B to another basis B', the same tableau $T(B')$ is obtained as if it was constructed directly with basis B'.

Problem 3: Programming Exercise (10 points)
Implement the simplex algorithm (Algorithm 2.15) for LP's of the form $\min\{\underline{c}\,\underline{x} : A\underline{x} \leq \underline{b}, \underline{x} \geq \underline{0}\}$ with a nonnegative right-hand-side vector $\underline{b} \geq \underline{0}$ in Matlab. (The programs should be submitted on a floppy disk.)
Apply your algorithm to the following problems:

(a)

$$\begin{array}{lrcll} \max & \displaystyle\sum_{i=1}^{5} x_i & & & \\ \text{s.t.} & \displaystyle\sum_{i=1}^{5} \frac{1}{i+k}\, x_i & \leq & \displaystyle\sum_{i=1}^{5} \frac{1}{i+k} & \forall\, k = 1, \ldots, 5 \\ & x_i & \geq & 0 & \forall\, i = 1, \ldots, 5. \end{array}$$

(b)

$$\begin{array}{lrcrcrcl} \max & 5x_1 & + & 4x_2 & + & 16x_3 & & \\ \text{s.t.} & 2x_1 & + & x_2 & + & 3x_3 & \leq & 5 \\ & x_1 & + & x_2 & + & 5x_3 & \leq & 3 \\ & & & & & x_1, x_2, x_3 & \geq & 0. \end{array}$$

Examine the impact of the value of the right-hand-side vector $\underline{b}$ on the optimal solution of Problem 3(b) by increasing or decreasing its coefficients slightly.

You can (if you wish) compare your solutions with those obtained by a simplex solver available on the internet, for example, at
http://www.mcs.anl.gov/home/otc/Guide/CaseStudies/simplex/ (start with "Try me").