Universität Erlangen-Nürnberg
Naturwissenschaftliche Fakultät I
Prof. Dr. K. Klamroth

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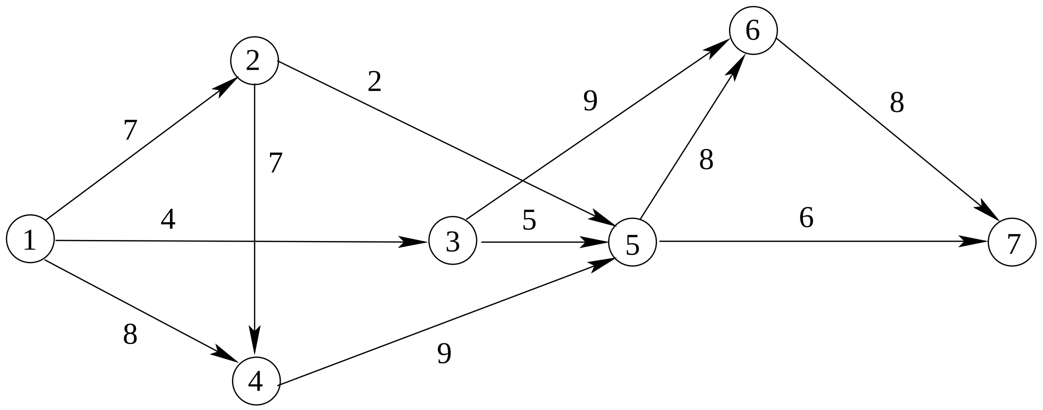
Linear and Network Optimization
Exercise 13

Please return your solutions by Tuesday, July 8^{th}, 10:00 a.m., in the mailbox No. 5.

Problem 1 (5 points)
Let V^+ and V^- be two nonempty disjoint node sets in G. Describe a method for determining the maximum number of arc-disjoint paths from V^+ to V^- (i.e., each path can start at any node in V^+ and can end at any node in V^-). What is the implication of the max-flow min-cut theorem in this case?

Problem 2 (7 points)

a) Find the maximum flow by applying the labeling algorithm of Ford & Fulkerson (Algorithm 7.18).



You may use the starting flow $\underline{x}$ with $x_{12} = x_{24} = x_{45} = x_{56} = x_{67} = 7$.

b) What is the complexity of this algorithm?

Problem 3 (8 points)
Optimal storage policy for libraries (Evans [1984]). A library facing insufficient primary storage space for its collection is considering the possibility of using secondary facilities, such as closed stacks or remote locations, to store portions of its collection. These options are preferred to an expensive expansion of primary storage. Each secondary storage facility has limited capacity and a particular access cost for retrieving information. Through appropriate data collection, we can determine the usage rates for the information needs of the users. Let b_j denote the capacity of storage facility j and v_j denote the access cost per unit item from this facility. In addition, let a_i denote the number of items of a particular class i requiring storage and let u_i denote the expected rate (per unit time) that we will need to retrieve books from this class. Our goal is to store the books in a way that will minimize the expected retrieval cost.

a) Show how to formulate the problem of determining an optimal policy as a transportation problem. What is the special structure of this problem? (Transportation problems with this structure have become known as factored transportation problems.)

b) Show that the simple rule that repeatedly assigns items with the greatest retrievel rate to the storage facility with lowest access cost specifies an optimal solution of this library storage problem.