Universität Erlangen-Nürnberg
Naturwissenschaftliche Fakultät I
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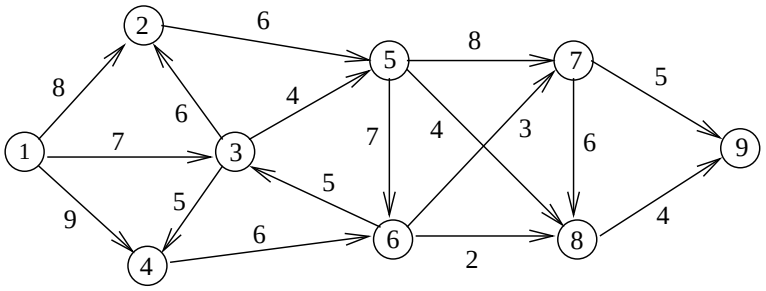
Linear and Network Optimization
Exercise 11

Please return your solutions by Tuesday, June 24th, 10:00 a.m., in the mailbox No. 5.

Problem 1 (5 points)
Apply the Dijkstra-Algorithm to the digraph G ($s = 1$):

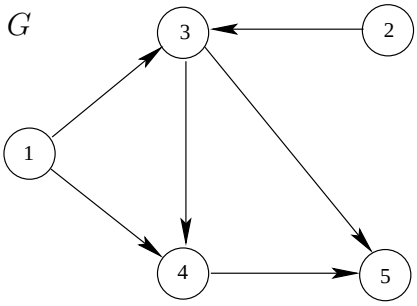


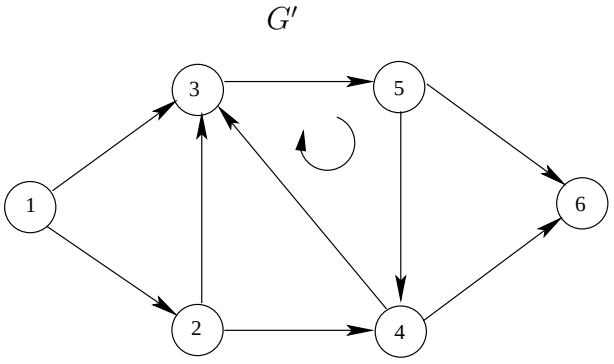
Specify the shortest path from node 1 to node 9 as well as the shortest dipath tree of G, using the predecssor labels.

Problem 2 (5 points)

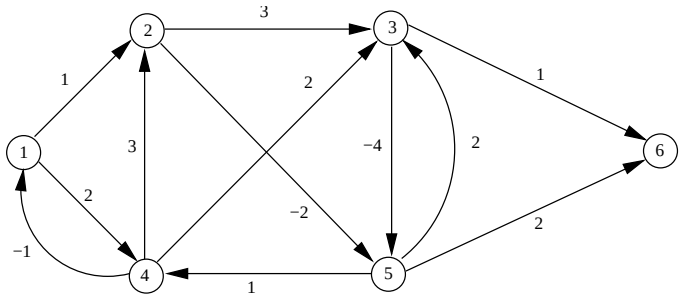
(a) Find an algorithmic proof for the following statement:
A digraph $G = (V, E)$ is acyclic if and only if its nodes can be numbered (with consecutive natural numbers $1, 2, 3, \ldots$) such that: number of tail(e) < number of head(e) for each $e \in E$.

(b) Apply your algorithm to check whether the digraphs G and G' are acyclic.





Problem 3 (5 points)
Use the label correcting algorithm to determine a shortest dipath from node 1 to node 6 in the following digraph:



Problem 4 (5 points)
Let $G = (V, E)$ be a digraph with source node $s \in V$ and sink node $t \in V$. Formulate the problem of finding a shortest dipath from s to t in G as an LP of the form:

$$\text{(P)} \quad \begin{array}{lrcll} \min & \underline{c}\,\underline{x} & & & \\ \text{s.t.} & A\,\underline{x} & = & \underline{b} & \\ & x_{ij} & \leq & 1 & \forall (i,j) \in E \\ & \underline{x} & \geq & \underline{0}, & \end{array}$$

where $\underline{x} \in \mathbb{R}^m$ and A is the incidence matrix of G.

(a) What values are represented by the vectors $\underline{c}$ and $\underline{b}$? What is the interpretation of a feasible solution $\underline{x} \in \mathbb{R}^m$ of (P)? (You may use an example to illustrate your findings.)

(b) Find the dual (D) of (P) and interpret your result in the light of the label correcting algorithm (see Theorem 6.12).

Can you give a geometric interpretation of an optimal solution of (D) for the case that $c_{ij} = c_{ji}\ \forall i, j \in V,\ i \neq j$?