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Naturwissenschaftliche Fakultät I
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Linear and Network Optimization
Exercise 1

Problem 1
Mother Courage and her children (1632):
Mother Courage owns a small business: She deals with gunpowder and spirits which she sells from a cart during the war. Her cart holds at most 240 l of spirits and at most 120 bags of gunpowder. Additionally, the capacity of the cart is restricted by a maximal load of 300 kg. We assume that both, 1 l of spirits and 1 bag of gunpowder, have a weight of 1 kg. Mother Courage has 3 children who are responsible for selling the gunpowder and the spirits. It takes them 10 min to sell 1 l of spirits and 40 min to sell 1 bag of gunpowder. The children should not work more than 100 hours per week (total). The profit for selling 1 bag of gunpowder is 20 Gulden and for selling 1 liter of spirit 10 Gulden. How many liters of spirits and how many bags of gunpowder should Mother Courage buy per week to maximize her profit?

(i) Find a linear programming model for the problem.
(ii) Solve the linear program graphically.

Problem 2
Solve the following LP graphically:

$$
\begin{array}{lrcrcr}
\max & x_1 & + & x_2 & & \\
\text{s.t.} & \frac{3}{5}x_1 & + & x_2 & \leq & \frac{18}{5} \\
 & x_2 & - & x_1 & \geq & -2 \\
 & 2x_1 & + & 2x_2 & \leq & 8 \\
 & & & x_2 & \leq & 3 \\
 & & & x_1, x_2 & \geq & 0
\end{array}
$$

Problem 3
Find a linear programming model for the following problem:

Chicken and Egg problem:
The owner of a small chicken farm must determine a laying and hatching program for 100 hens. There are currently 100 eggs in the henhouse, and the hens can either hatch existing eggs or lay new ones. In each 10-day period, a hen can either hatch 4 eggs or lay 12 new eggs. Chicken babies that are hatched can be sold for 60 cents each and every 30 days an egg dealer gives 10 cents each for the eggs accumulated to date. Eggs not being hatched in one period can be kept in a special incubator room for hatching in a later period. The problem is to determine how many hens should be hatching and how many should be laying in each of the next three 10-day periods so that total revenue is maximized.

Problem 4
Consider the problem of locating a new machine to an existing layout consisting of four machines at the following coordinates:

$$\begin{pmatrix} 3 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ -3 \end{pmatrix}, \quad \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 4 \end{pmatrix}.$$

Let the coordinates of the new machine be $(x_1, x_2)^T$. Formulate the problem of finding an optimal location as a linear program for each of the following cases.

(a) The sum of the distances from the new machine to the four machines is minimized. Use the rectangular street distance (i.e., the l_1 metric); for example, the distance from $(x_1, x_2)^T$ to the first machine located at $(3, 0)^T$ is $|x_1 - 3| + |x_2|$.

(b) Because of various amounts of flow between the new machine and the existing machines, reformulate the problem such that the sum of the weighted distances is minimized, where the weights corresponding to the four machines are 5, 7, 3, and 1 respectively.

(c) In order to avoid congestion, suppose that the new machine must be located in the square $\{(x_1, x_2)^T : -1 \leq x_1 \leq 2,\ 0 \leq x_2 \leq 1\}$. Formulate (a) and (b) with this added restriction.

(d) Suppose that the new machine must be located so that its distance from the first machine does not exceed $\frac{3}{2}$. Formulate the problem with this added restriction.

Problem 5
Write the following LP's in standard form:

$$\begin{array}{lrcrcl}
\max & 4x_1 & + & 5x_2 & - & x_3 \\
\text{s.t.} & x_1 & - & 4x_2 & \leq & 3 \\
& 8x_1 & + & x_3 & \geq & 1 \\
& x_2 & + & x_3 & = & 3 \\
& & & x_2 & \geq & 0 \\
& & & x_1, x_3 & \in & \mathbb{R}
\end{array}
\qquad
\begin{array}{lrcrcl}
\min & 3x_1 & + & x_2 & - & x_3 \\
\text{s.t.} & 2x_1 & + & x_2 & \leq & 3 \\
& & & x_1 & \geq & 7 \\
& 2x_1 & - & x_3 & = & 4 \\
& & & x_i & \in & \mathbb{R}, \quad i = 1, 2, 3
\end{array}$$