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**Algorithm 4.3:** Karmarkar's Projective Algorithm

(Input) • LP, with optimal objective value 0 and integer $A, \underline{b}, \underline{c}$, in the form

$$(LP) \qquad \begin{array}{rrcl} \min & \underline{c}\,\underline{x} & & \\ \text{s.t.} & A\,\underline{x} & = & \underline{0} \\ & \underline{e}\,\underline{x} & = & 1 \\ & \underline{x} & \geq & \underline{0}, \end{array}$$

- interior feasible point $\underline{x}^0$,
- large integer $L$,
  step length $0 < \alpha < 1$ and $r = \frac{1}{\sqrt{n(n-1)}}$.

(1) **Stopping criterion:**
If $\underline{c}\,\underline{x}^k < 2^{-L}$, goto Step (5).

(2) **Projective transformation:**
Transform (LP) to

$$(\overline{LP}) \qquad \begin{array}{rrcl} \min & \underline{c}\,D^k\,\underline{y} & & \\ \text{s.t.} & A\,D^k\,\underline{y} & = & \underline{0} \\ & \underline{e}\,\underline{y} & = & 1 \\ & \underline{y} & \geq & \underline{0} \end{array}$$

with $D^k := \begin{pmatrix} x_1^k & & 0 \\ & \ddots & \\ 0 & & x_n^k \end{pmatrix}$.

Let $\bar{\underline{c}} := \underline{c}\,D^k$, $\bar{A} := \begin{pmatrix} A\,D^k \\ 1 \cdots 1 \end{pmatrix}$, $\bar{\underline{b}} := \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $\underline{y}^k := (\frac{1}{n}, \ldots, \frac{1}{n})^T$.

(3) **Move in direction of steepest descent:**
Determine $\bar{\underline{c}}_p := (I - \bar{A}^T(\bar{A}\bar{A}^T)^{-1}\bar{A})\,\bar{\underline{c}}^T$, and set

$$\underline{y}^{k+1} := \underline{y}^k - \alpha\, r\, \frac{\bar{\underline{c}}_p}{||\bar{\underline{c}}_p||}.$$

(4) **Inverse transformation:**
Determine

$$\underline{x}^{k+1} := \frac{D^k\,\underline{y}^{k+1}}{\underline{e}\,D^k\,\underline{y}^{k+1}}.$$

Set $k := k + 1$ and goto Step (1).

(5) **Optimal rounding:**
Given $\underline{x}^k$, determine a basic feasible solution $\underline{x}^*$ with $\underline{c}\,\underline{x}^* \leq \underline{c}\,\underline{x}^k < 2^{-L}$ using a purification scheme and STOP.