Universität Erlangen-Nürnberg
Naturwissenschaftliche Fakultät I
Prof. Dr. K. Klamroth

Algorithm 3.12: Primal-Dual Simplex Method

(Input) (LP) $\min\{\underline{c}\,\underline{x} : A\underline{x} = \underline{b} \geq \underline{0},\ \underline{x} \geq \underline{0}\}$,
dual feasible solution $\underline{\pi}$.

(1) $J := \{j : A_j^T \underline{\pi}^T = c_j\}$.

(2) Solve the **reduced primal LP**

$$
\textbf{(RP)} \qquad
\begin{array}{lrcl}
\min & w = \sum_{i=1}^{m} \hat{x}_i & & \\
\text{s.t.} & \sum_{j \in J} x_j \cdot A_j + \hat{\underline{x}} & = & \underline{b} \\
 & x_j, \hat{x}_i & \geq & 0 \quad \forall\, i, j
\end{array}
$$

or its dual

$$
\textbf{(RD)} \qquad
\begin{array}{lrcll}
\max & v = \underline{b}^T \underline{\alpha}^T & & & \\
\text{s.t.} & A_j^T \underline{\alpha}^T & \leq & 0 & \forall\, j \in J \\
 & \alpha_i & \leq & 1 & \forall\, i = 1, \ldots, m \\
 & \alpha_i & \gtreqless & 0 & \forall\, i = 1, \ldots, m
\end{array}
$$

<u>Remark:</u> The optimal basis of the previous iteration can be used as a feasible starting basis in (RP).

(3) If $w_{\text{opt}} = v_{\text{opt}} = 0$ for the optimal objective value
(STOP), for an optimal solution $(x_j)_{j \in J}$ of (RP) set
$x_j := 0 \ \ \forall\, j \notin J$;
$\underline{x} = (x_j)_{j=1}^{n}$ is an optimal solution of (P).
Otherwise determine a dual optimal solution $\underline{\alpha}_{\text{opt}}$ of (RD).

(4) If $A_j^T \underline{\alpha}_{\text{opt}}^T \leq 0 \quad \forall\, j \notin J$,
(STOP), (D) is unbounded, i.e. (P) is infeasible.
Otherwise set

$$
\begin{aligned}
\delta &:= \min \left\{ \frac{c_j - A_j^T \underline{\pi}^T}{A_j^T \underline{\alpha}_{\text{opt}}^T} : j \notin J,\ A_j^T \underline{\alpha}_{\text{opt}}^T > 0 \right\} \\
\underline{\pi} &:= \underline{\pi} + \delta \cdot \underline{\alpha}_{\text{opt}}
\end{aligned}
$$

and goto Step (1).