Universität Erlangen-Nürnberg
Naturwissenschaftliche Fakultät I
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Prof. Dr. K. Klamroth
Eva Bijick

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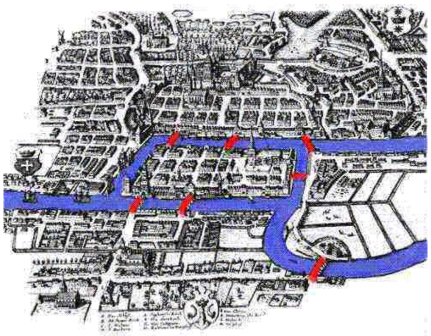
Network Optimization
Exercise 1

Problem 1
Paragraph problem. The well-known document processing program LaTeX uses an optimization procedure to decompose a paragraph into several lines so that when lines are left- and right-adjusted, the appearance of the paragraph will be the most attractive. Suppose that a paragraph consists of n words and that each word is assigned a sequence number. Let c_{ij} denote the attractiveness of a line if it begins with the word i and ends with the word $j-1$. The program LaTeX uses formulas to compute the value of each c_{ij}. Given the c_{ij}'s, show how to formulate the problem of decomposing the paragraph into several lines of text in order to maximize the total attractiveness (of all lines) as a shortest path problem.

Problem 2
Seat-sharing problem. Several families are planning a shared car trip on scenic drives in the Alpes. To minimize the possibility of any quarrels, they want to assign individuals to cars so that no two members of a family are in the same car. Formulate this problem as a network flow problem.

Problem 3
Bridges of Königsberg. The first paper on graph theory was written by Leonhard Euler in 1736. In this paper, he started with the following mathematical puzzle: The city of Königsberg has seven bridges, arranged as shown below. Is it possible to start at some place in the city, cross every bridge exactly once, and return to the starting place? Either specify such a tour or prove that it is impossible to do so.



Problem 4
Let $\mathcal{N}$ denote the node-arc incidence matrix of an undirected graph and let $\mathcal{N}^T$ denote its transpose. Let "$\cdot$" denote the operation of taking a product of two matrices. Show how to interpret the diagonal elements of $\mathcal{N} \cdot \mathcal{N}^T$.

Problem 5
Let $\mathcal{H}$ be the node-node adjacency matrix of a directed graph $G = (N, A)$. Let $\mathcal{H}^T$ be the transpose of $\mathcal{H}$, and let G^T be the graph corresponding to $\mathcal{H}^T$. How is the graph G^T related to G?