Location Analysis, Handout 2: Restricted Single Facility Weber Problems, $1/P/R/l_p/\sum$

Boundary Search Algorithm for $1/P/R/l_p/\sum$

Input: Existing facilities $a_1, \ldots, a_n \in I\!R^2$; weights $w_1, \ldots, w_n > 0$, forbidden region $R = R_1 \dot{\cup} \cdots \dot{\cup} R_K \subset I\!R^2$ with R_k connected and closed, $\partial R_k = \{g_k(t) \,:\, t \in [0;1]\}$, $k = 1, \ldots, K$.

Step 1: Determine an (approximate) optimal location X^* for $1/P/\bullet/l_p/\sum$.

Step 2: If $X^* \in I\!R^2 \setminus \mathrm{int}(R)$, set $X^*_R := X^*$, STOP.

Step 3: Determine $k \in \{1, \ldots, K\}$ with $X^* \in \mathrm{int}(R_k)$.

Step 4: Solve the 1-dimensional optimization problem

$$\min\{W(g_k(t)) \,:\, t \in [0;1]\}$$

and set $X^*_R := g_k(t^*)$, STOP.

Output: Optimal solution (or approximation) X^*_R of $1/P/R/l_p/\sum$.

Algorithm for $1/P/(R = 1\,polyhedron)/l_2^2/\sum$

Input: Existing facilities $a_1, \ldots, a_n \in I\!R^2$; weights $w_1, \ldots, w_n > 0$, forbidden region $R = \{X \in I\!R^2 \,:\, B \cdot X \leq b\} \subset I\!R^2$, where $B = (B^1, B^2)$ is an $L \times 2$-matrix and $b = (b_1, \ldots, b_L)^T$ is an L-vector.

Step 1: Determine

$$x_k^* = \frac{\sum_{j=1}^n w_j a_{jk}}{\sum_{j=1}^n w_j}, \quad k = 1, 2$$

and set $X^* := (x_1^*, x_2^*)^T$.

Step 2: If $B \cdot X^* \geq b$, set $\mathcal{X}_R^* := \{X^*\}$, STOP.

Step 3: Determine the projections P_l of X^* onto the lines

$$B_l^1 x_1 + B_l^2 x_2 = b_l, \quad l = 1, \ldots, L.$$

Step 4: Set $\mathcal{X}_R^* := \{P_{l^*} \,:\, l_2(P_{l^*}, X^*) = \min_{l \in \{1,\ldots,L\}} l_2(P_l, X^*)\}$, STOP.

Output: Optimal solutions $\mathcal{X}_R^*$ of $1/P/(R = 1\,polyhedron)/l_2^2/\sum$.

Construction Line Algorithm for $1/P/R\,convex/l_1/\sum$

Input: Existing facilities $a_1, \ldots, a_n \in I\!R^2$; weights $w_1, \ldots, w_n > 0$, convex forbidden region $R \subset I\!R^2$.

Step 1: Determine the set of optimal solutions $\mathcal{X}^*$ of $1/P/\bullet/l_1/\sum$.

Step 2: If $\mathcal{X}^* \cap (I\!R^2 \setminus \mathrm{int}(R)) \neq \emptyset$, set $\mathcal{X}_R^* := \mathcal{X}^* \cap (I\!R^2 \setminus \mathrm{int}(R))$, STOP.

Step 3: Determine the construction line grid

$$\begin{aligned} \mathcal{G}_{l_1} \quad := \quad & \{(x_1, x_2)^T \in I\!R^2 \,:\, x_1 = a_{j1},\, j \in \{1, \ldots, n\}\} \\ & \cup \{(x_1, x_2)^T \in I\!R^2 \,:\, x_2 = a_{j2},\, j \in \{1, \ldots, n\}\} \end{aligned}$$

and the intersection points $\{Y_1, \ldots, Y_L\} := \mathcal{G}_{l_1} \cap \partial R$.

Step 4: Set $X_R^* := \mathrm{argmin}\{W(Y_1), \ldots, W(Y_L)\}$.

Step 5: If $\mathcal{X}_R^*$ is sought, determine the level set $L_=(W(X_R^*))$ and set $\mathcal{X}_R^* := (L_=(W(X_R^*)) \cap \partial R)$, STOP.

Output: Optimal solution(s) X_R^* ($\mathcal{X}_R^*$) of $1/P/R\,convex/l_1/\sum$.