Institute of Applied Mathematics
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December 2, 2004

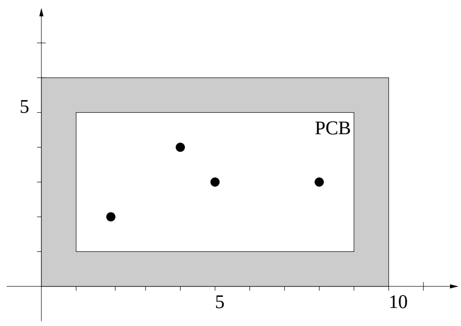
Location Analysis
WS 2004/2005
Homework 3

To be discussed in the tutorial on December 9, 2004.

9. Find an algorithm to solve problems of the type $1/P/R/l_2^2/\sum$ where the forbidden region R is a non-convex polyhedron given by its corner points $(y_1, \ldots, y_L)$.

   Apply the algorithm to an example problem with existing facility locations $a_1 = (1;1)$, $a_2 = (1;4)$, $a_3 = (2;1)$, $a_4 = (4;1)$, $a_5 = (4;4)$, weights $w_1 = 2$, $w_2 = 1$, $w_3 = 1$, $w_4 = 2$, $w_5 = 4$ and forbidden region R with corner points (in clockwise order) $y_1 = (0;0.4)$, $y_2 = (0;5)$, $y_3 = (7;5)$, $y_4 = (7;3)$, $y_5 = (5;3)$, $y_6 = (5;0.4)$.

10. Consider a PCB of the size $[1;9] \times [1;5]$. A circular part has to be placed at each of the locations $(2;2)$, $(4;4)$, $(5;3)$ and $(8;3)$. This is done using a robot arm which needs a time proportional to $\max\{|x_1 - y_1|, |x_2 - y_2|\}$ to move from a point $X = (x_1;x_2)$ to another point $Y = (y_1;y_2)$. Find an optimal location for a container containing the circular parts that has a security distance of 1 to the PCB using the boundary search algorithm. It is sufficient to search for the solution along the upper boundary of the forbidden region, that is, along the segment $\{(x_1;x_2) : x_1 \in [0;10], x_2 = 6\}$.



11. Prove Lemma 3.21:
   Let $R \subset \mathbb{R}^2$ be closed and convex, and let $\mathcal{X}^* \subseteq \text{int}(R)$ for $1/P/\bullet/l_1/\sum$. Then there is an optimal solution X_R^* of $1/P/R/l_1/\sum$ such that

$$X_R^* \in \partial R \quad \text{and} \quad X_R^* \in \mathcal{G}_{l_1}$$

   where

$$\begin{aligned} \mathcal{G}_{l_1} &= \left\{(x_1, x_2) \in \mathbb{R}^2 : x_1 = a_{j_1}, j \in \{1, \ldots, n\}\right\} \\ &\quad \cup \left\{(x_1, x_2) \in \mathbb{R}^2 : x_2 = a_{j_2}, j \in \{1, \ldots, n\}\right\} \end{aligned}$$

   is the construction line grid with respect to the existing facility locations at $a_1, \ldots, a_n$.

12. Solve the problem from exercise 10. using the construction line algorithm for $1/P/R/l_1/\sum$.