Institute of Applied Mathematics
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Location Analysis
WS 2004/2005
Homework 2

To be discussed in the tutorial on November 25, 2004.

5. a) Show that the optimal solution $X^*_{l_2^2}$ of a problem of type $1/P/\bullet/l_2^2/\sum$ with existing facility locations at $a_1, \ldots, a_n$ is always located in conv $(a_1, \ldots, a_n)$, the convex hull of the existing facilities.

   b) Show that the same also holds for problems of type $1/P/\bullet/l_2/\sum$.

6. Justify the following properties of problems of type $1/S/\bullet/A/\sum$:

   a) A point on S is a minimizer for $\sum_{j=1}^{n} w_j A(X, a_j)$ if, and only if, its antipode is a maximizer.

   b) A point and its antipode with equal weights can be added to the problem without a change in the optimal location of the facility.

   c) A point with weight w_j can be replaced by its antipode with weight $-w_j$ without changing the optimal location of the facility.

   d) Every problem can be transformed to an equivalent problem that has only positive weights.

7. Derive the equations

$$\tan x_2 = \frac{\sum_{j=1}^{n} \left(w_j \cos a_{j1} \sin a_{j2}\right) / \sin A(X, a_j)}{\sum_{j=1}^{n} \left(w_j \cos a_{j1} \cos a_{j2}\right) / \sin A(X, a_j)}$$

$$\frac{\tan x_1}{\sin x_2} = \frac{\sum_{j=1}^{n} \left(w_j \sin a_{j1}\right) / \sin A(X, a_j)}{\sum_{j=1}^{n} \left(w_j \cos a_{j1} \sin a_{j2}\right) / \sin A(X, a_j)}$$

   from the optimality conditions $\frac{\partial W(X)}{\partial x_1} = \frac{\partial W(X)}{\partial x_2} = 0$.

8. Show that Lemma 3.5 is "tight" in the sense that an example with two or more local minima can be constructed if the radius is $\pi/4 + \varepsilon$, where $\varepsilon > 0$.