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Location Analysis
WS 2004/2005
Homework 1

To be discussed in the tutorial on November 4, 2004.

1. Jack Smooth, an industrial sales and service representative, wants to find a new office location. His business involves several trips a week to seven factories in a large suburban area. From a travel expense diary, he has made up some averages based on several months' experience. He has marked the location of each plant on a map and, using the left and bottom borders of the map as coordinate axes, has given a location to each plant. This information is contained in the following table:

Customer	Average Number of Weekly Trips	Location
A	5	(5;20)
B	7	(18;8)
C	2	(22;16)
D	3	(14;17)
E	6	(7;2)
F	1	(5;15)
G	5	(12;4)

Jack Smooth's objective is to find an office location that minimizes total travel distance. He considers rectangular distances to be the best representation of the actual travel distances involved.

a) Solve Jack Smooth's office location problem.

b) Plot $W_k(x_k)$ against x_k, $k = 1, 2$.

c) Graph the problem setting together with the optimal solution and some level curves of the objective function.

2. Why can the algorithm for the solution of $1/P/\bullet/l_1/\sum$ not be applied if the assumption that $w_j > 0$, $j = 1, \ldots, n$ is relaxed to $w_j \neq 0$, $j = 1, \ldots, n$?

3. Show that $W(X) = \sum_{j=1}^{n} w_j(l_2(X, a_j))^2$ is strictly convex.

4. The Varignon frame is a mechanical analog for the minimization of $W(X)$ in $1/P/\bullet/l_2/\sum$. It consists of a board with holes drilled in it to correspond to fixed facility locations. A string is passed through each hole j, and the ends are tied together in a knot on top of the board. Under the board a weight is attached to each string, and the weight on string j is proportional to the demand w_j of existing facility a_j. In the absence of friction and tangled strings, the knot will come to rest at the optimum new facility location.

   This analog can be analyzed in terms of forces acting to move the knot. Assume that the knot is in equilibrium. In the figure below, for example, the weight w_4 acts with a force w_4, which can be broken up into the orthogonal components w_{41} and w_{42}, where

$$w_{41} = w_4 \cos\theta_4 \qquad \text{and} \qquad w_{42} = w_4 \sin\theta_4.$$

   Note that $w_4^2 = w_{41}^2 + w_{42}^2$.

   a) Show that
   $$\cos\theta_4 = \frac{a_{41} - x_1}{l_2(X, a_4)} \qquad \text{and} \qquad \sin\theta_4 = \frac{a_{42} - x_2}{l_2(X, a_4)},$$
   where $x = (x_1, x_2)$ is the equilibrium position.

   b) Show that a balance of x_1-direction force components and a balance of x_2-direction force components is, in general, equivalent to $\frac{\partial W(X)}{\partial x_k} = 0$ for $k = 1, 2$.

   c) Explain the optimality condition at an existing facility a_r, $CR_r \leq w_r$, in terms of forces.