Location Analysis, Handout 5:
Set Covering Problems, $\#/D/\bullet/\bullet/\sum_{\text{cov}}$

Algorithm 6.8: Branch and Bound Algorithm for $\#/D/\bullet/\bullet/\sum_{\text{cov}}$

Input: Finite set of demand nodes I, finite set of candidate sites J, $a_{ij} \in \{0,1\}$ $\forall i \in I$, $j \in J$.

Step 1: *Initial solution:*
Apply the reduction rules 1, 2a and 2b to obtain a reduced IP-formulation of the problem.
Let $\bar{z}$ be an upper bound on the optimal objective value (sufficiently large).

Step 2: *Initial relaxation:*
Solve the LP-relaxation of the problem determined in Step 1 and let $\underline{z}_1$ be its objective value (lower bound).
Node P_1 of the Branch and Bound tree represents the present problem and is the only live node.

Step 3: *Branch and Bound procedure:*
Does any live node exist in the solution tree?
- If yes: Choose a live node P_k (e.g., the node with the best lower bound $\underline{z}_k$), and goto Step 4.
- If no: The best known feasible solution is optimal. (If no such solution is known, the problem is infeasible.)

Step 4: Is the solution represented by node P_k feasible (for the original problem)?
- If yes: (STOP), the solution in node P_k is optimal.
- If no: Goto Step 5.

Step 5: *Branching:*
Select a decision variable X whose value in the relaxed problem at node P_k is $X = \gamma \notin \mathbb{N}$ but must be integer in a feasible solution.
Branch from node P_k to nodes P_{s+1}, P_{s+2}, so that, in addition to the constraints added earlier, at node P_{s+1} we set $X \leq \lfloor \gamma \rfloor$ and at node P_{s+2} we set $X \geq \lceil \gamma \rceil$.

Step 6: *Bounding:*
For each node P_{s+k}, $k = 1, 2$, do:
Solve the LP relaxation including the constraints added in Step 5.
Let its objective value be $\underline{z}_{s+k}$.
- If $\underline{z}_{s+k} \geq \bar{z}$, fathom node P_{s+k}.
- If $\underline{z}_{s+k} < \bar{z}$ and the solution is feasible, set $\bar{z} := \underline{z}_{s+k}$ and fathom node P_{s+k}.
- If $\underline{z}_{s+k} < \bar{z}$ and the solution is infeasible, the node P_{s+k} is live.

Goto Step 3.

Output: Optimal solution of $\#/D/\bullet/\bullet/\sum_{\text{cov}}$ with objective value $\bar{z}$.