Model Development, Uncertainty Quantification, and Control Design for Nonlinear Smart Material Systems

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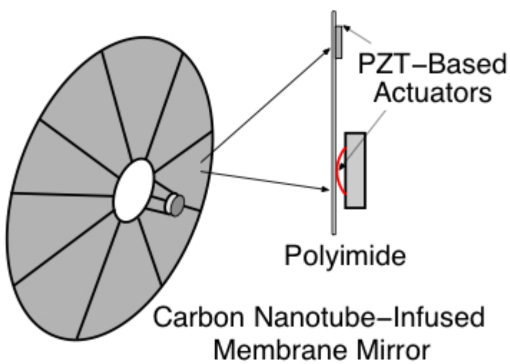
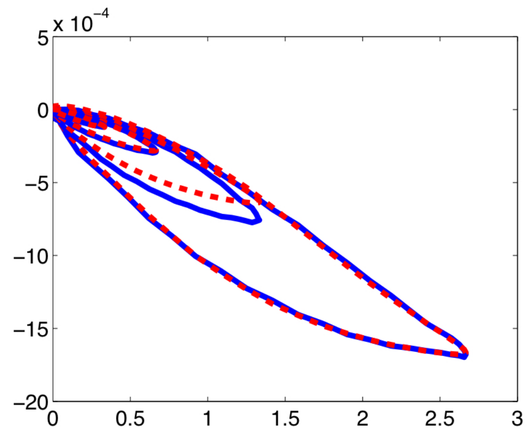
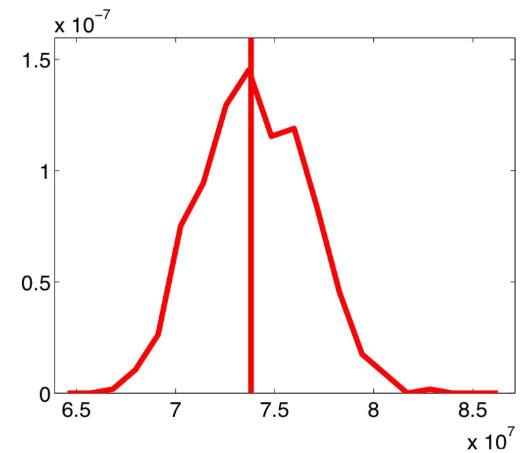


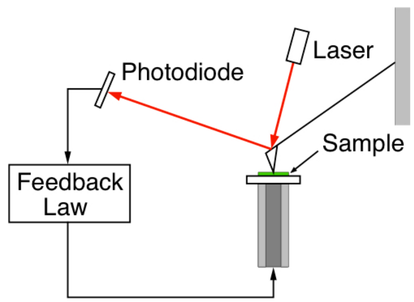




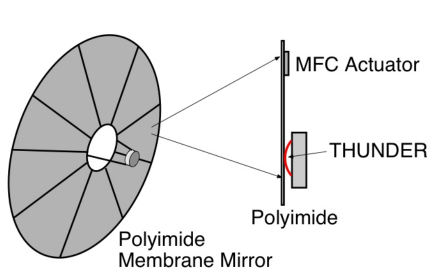
Research Support: Air Force grant AFOSR FA9550-08-1-0348

Applications

Ferroelectric (e.g., PZT)



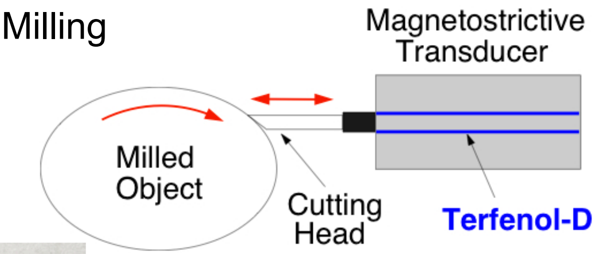
High Speed Nano-positioning



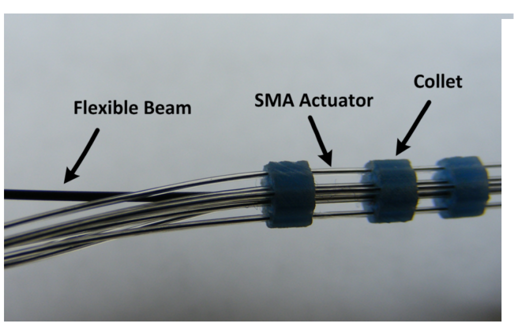
Membrane mirrors/ antennas

Ferromagnetic (e.g., Terfenol-D)

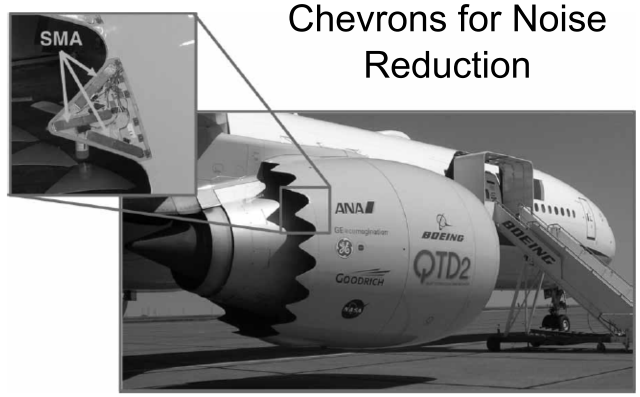
High Speed Milling



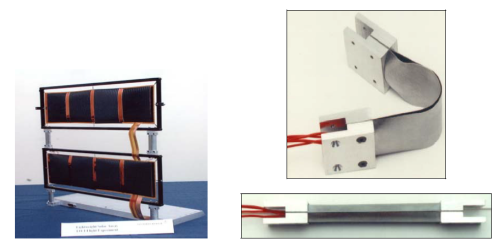
Ferroelastic (e.g., Shape Memory Alloy)



Catheters for Laser Ablation

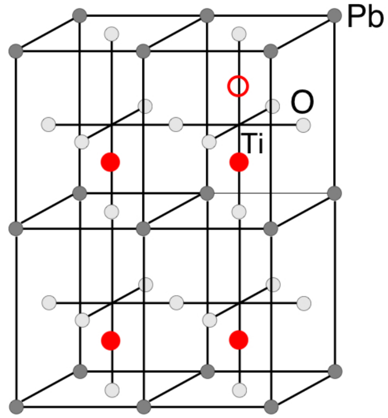


Chevrons for Noise Reduction

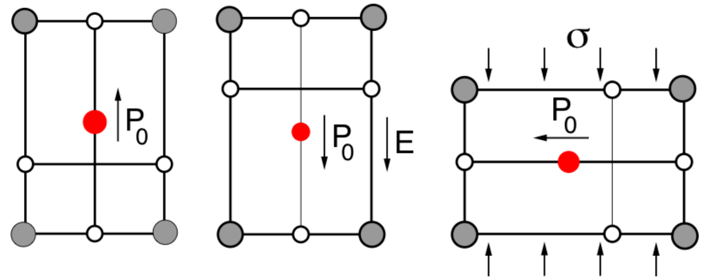


SMA Hinges for Solar Arrays

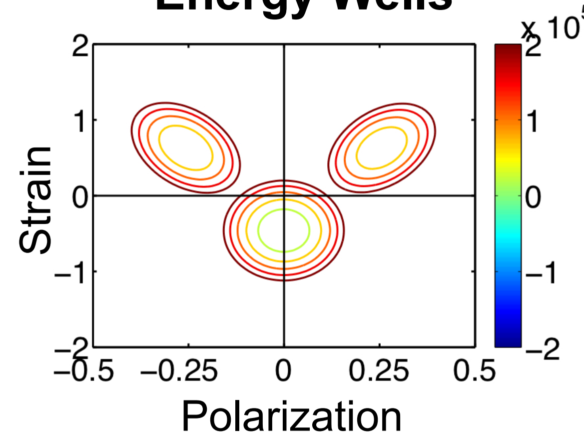
Ferroelectric Model Development -- Mesoscopic Level



Three Primary Variants in 1-D



Energy Wells



Helmholtz Energy Density: $\alpha = \pm 180, 90$

$$\psi_\alpha(P,\varepsilon) = \frac{1}{2}\eta_\alpha^\varepsilon (P - P_R^\alpha)^2 + \frac{1}{2}c_\alpha^P(\varepsilon - \varepsilon_R^\alpha)^2 + h_\alpha(P - P_R^\alpha)(\varepsilon - \varepsilon_R^\alpha)$$

Gibbs Energy Density:

$$G_\alpha(E,\sigma;P,\varepsilon) = \psi_\alpha(P,\varepsilon) - EP - \sigma\varepsilon$$

Thermodynamic Equilibria: $\dfrac{\partial G}{\partial P} = 0 \; , \; \dfrac{\partial G}{\partial \varepsilon} = 0$

$$P^\alpha = P_R^\alpha + \chi_\alpha^\sigma E + d_\alpha \sigma$$

$$\varepsilon^\alpha = \varepsilon_R^\alpha + d_\alpha E + s_\alpha^E \sigma$$

Note:

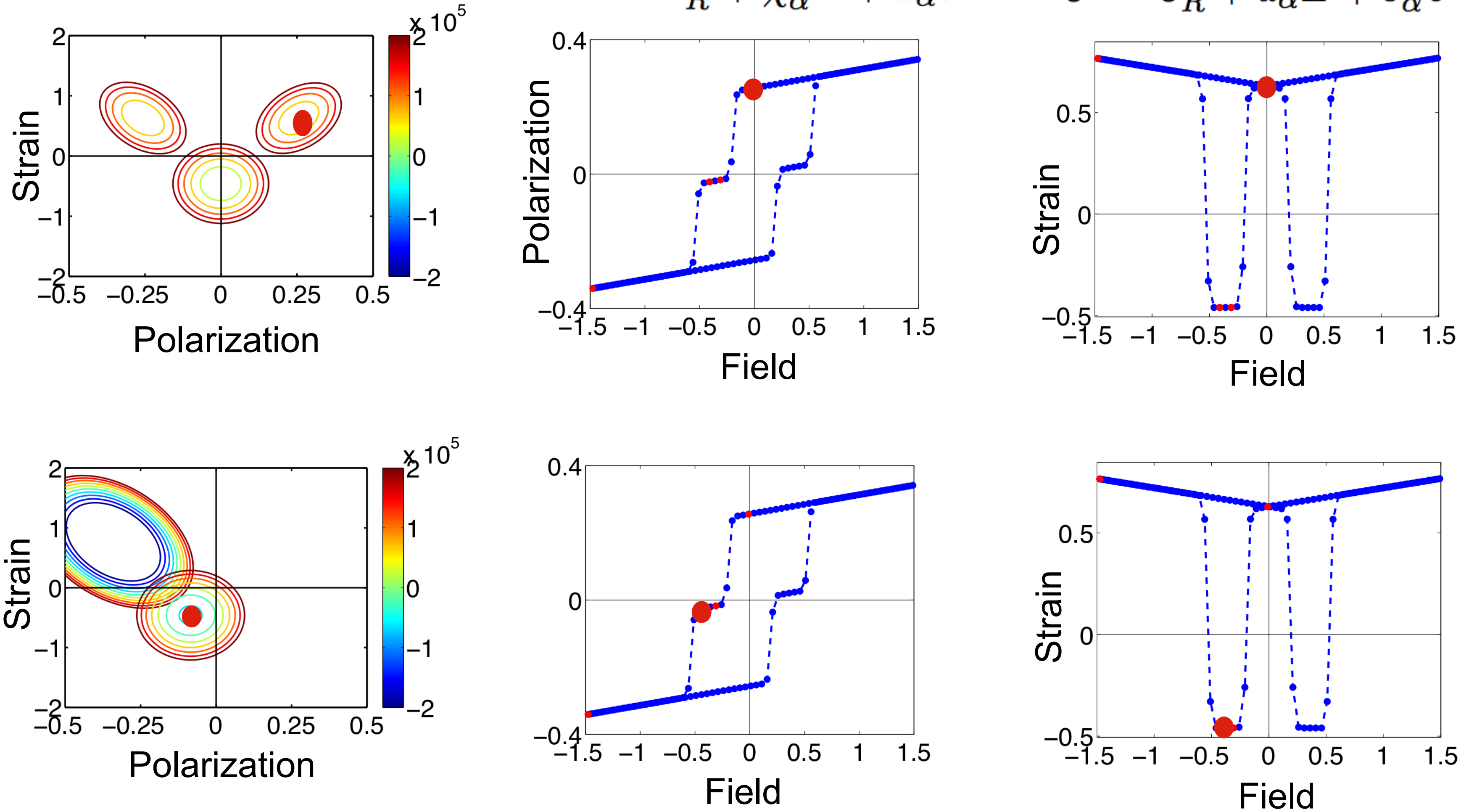
- Linear in each well
- Hysteresis, nonlinearities due to switching between wells

Model Development -- Mesoscopic Level

Thermodynamic Behavior:

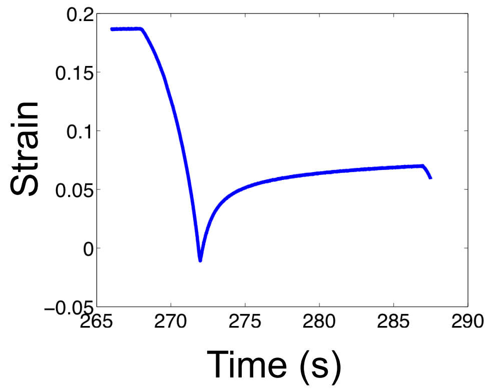
$$P^\alpha = P_R^\alpha + \chi_\alpha^\sigma E + d_\alpha \sigma$$

$$\varepsilon^\alpha = \varepsilon_R^\alpha + d_\alpha E + s_\alpha^E \sigma$$



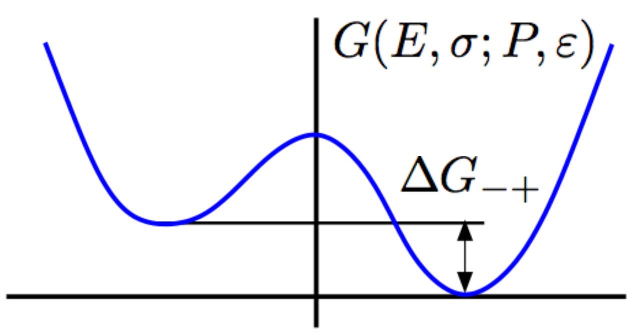
Model Development -- Mesoscopic Level

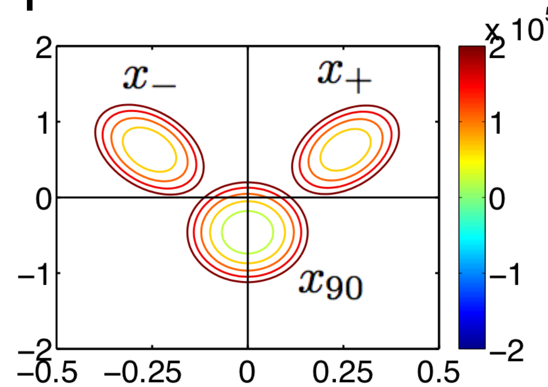
Problem: Kinetics produce creep



Solution: Theory of thermally activated processes. E.g., minimize

$$G = \phi(p) - K(T) - TS - Ep$$
$$= \phi(p) - K(T) - Tk \ln \frac{N_-}{\prod_{p \in S_-} N_p} - Ep$$





Dipole Fractions:

$$\dot{x}_- = -p_{-90}x_- + p_{90-}x_{90}$$
$$\dot{x}_{90} = p_{-90}x_- - (p_{90-} + p_{90+})x_{90} + p_{+90}x_+$$
$$\dot{x}_+ = p_{90+}x_{90} - p_{+90}x_+$$

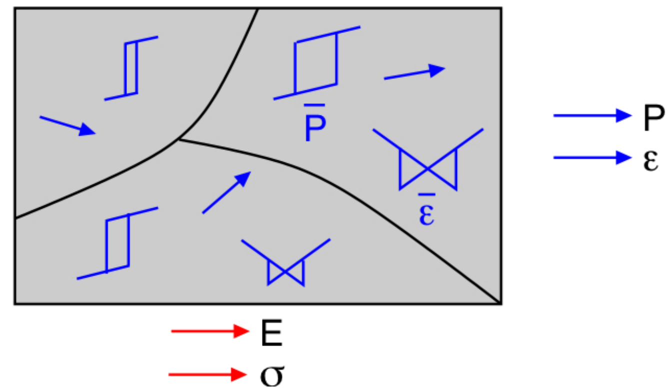
Transition Likelihoods:

$$p_{\alpha\beta} = \frac{1}{\tau} e^{-\Delta G_{\alpha\beta} V / kT}$$

Polarization and Strain Kernels:

$$\overline{P} = \sum_{\alpha = \pm, 90} x_\alpha P^\alpha \quad , \quad \overline{\varepsilon} = \sum_{\alpha = \pm, 90} x_\alpha \varepsilon^\alpha$$

Model Development -- Macroscopic Level



Ferroelectric Materials:

- Incorporate grains, polycrystallinity, variable interaction fields

Homogenized Energy Model (HEM):

$$\varepsilon(E(t),\sigma(t);x_+) = \int_0^\infty \int_{-\infty}^\infty \bar{\varepsilon}(E(t)+E_I;F_c)\nu_I(E_I)\nu_c(F_c)dE_I dF_c$$

Interaction, coercive field densities

Note:

$$\begin{aligned} \bar{\varepsilon} &= \sum_{\alpha=\pm,90} x_\alpha(E,\sigma)\left[\varepsilon_R^\alpha + d_\alpha E + s^E\sigma\right] \\ &= S^E\sigma + \bar{d}(E,\sigma)E + \bar{\varepsilon}_{irr}(E,\sigma) \end{aligned}$$

Constitutive Relation:

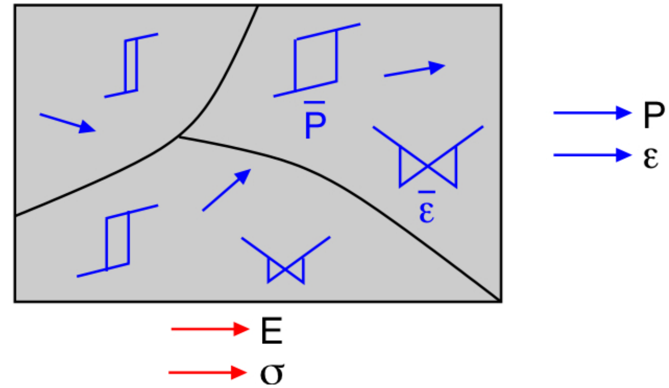
$$\varepsilon(E,\sigma) = s^E\sigma + d(E,\sigma_0)E + \varepsilon_{irr}(E,\sigma_0)$$

$$\Rightarrow \sigma(E,\varepsilon) = c^E\varepsilon - e(E,\sigma_0)E - c^E\varepsilon_{irr}(E,\sigma_0)$$

Examples:

- Beams, shells, structural-acoustic systems

Model Development -- Macroscopic Level



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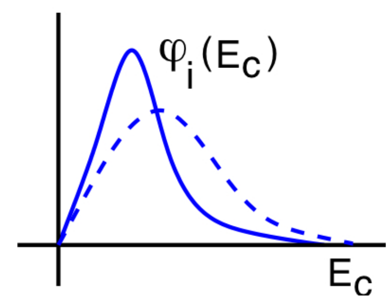
Interaction, coercive field densities

Density Representations:

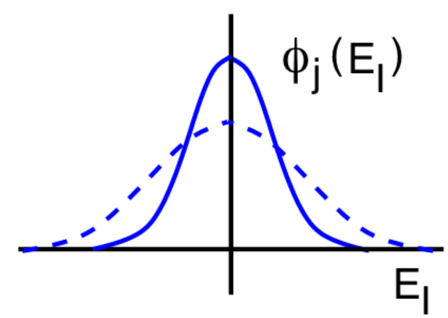
$$\nu_I(E_I) = c_2 \sum_{j=1}^{N_\beta} \beta_j \phi_j(E_I) \quad , \quad \nu_c(E_c) = c_1 \sum_{i=1}^{N_\alpha} \alpha_i \varphi_i(E_c)$$



Basis Choices:

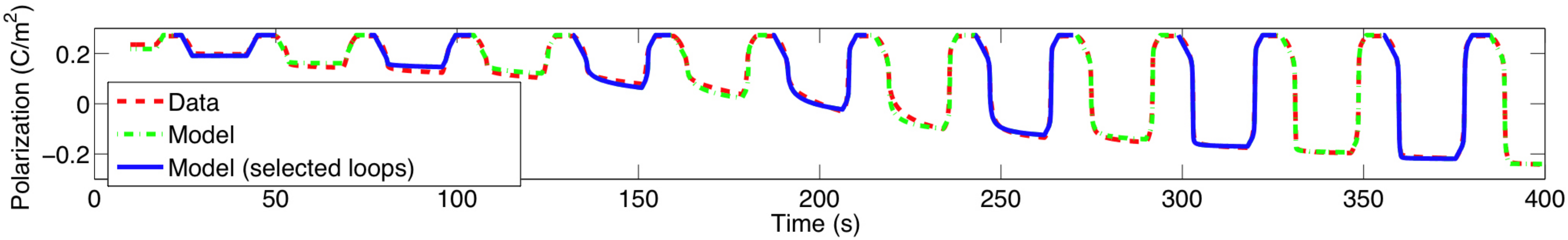
$$\phi_j(E_I) = \frac{1}{\sigma_I^j\sqrt{2\pi}} e^{-E_I^2/2(\sigma_I^j)^2}$$

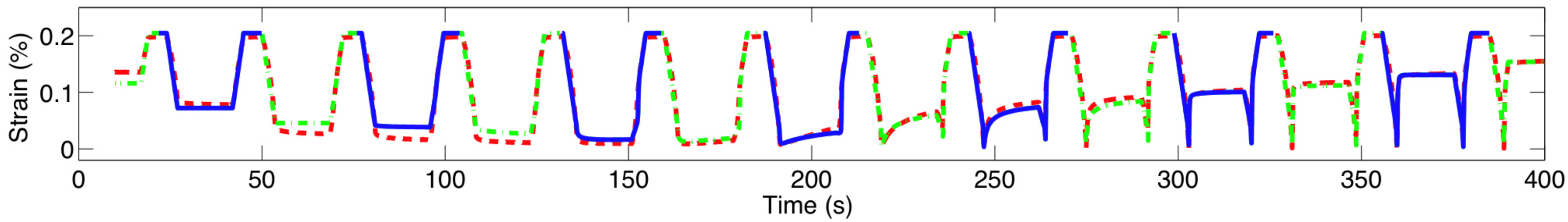
$$\varphi_i(E_c) = \frac{1}{\sigma_c^i E_c\sqrt{2\pi}} e^{-[\ln(E_c)-\mu_c^i]^2/2(\sigma_c^i)2}$$

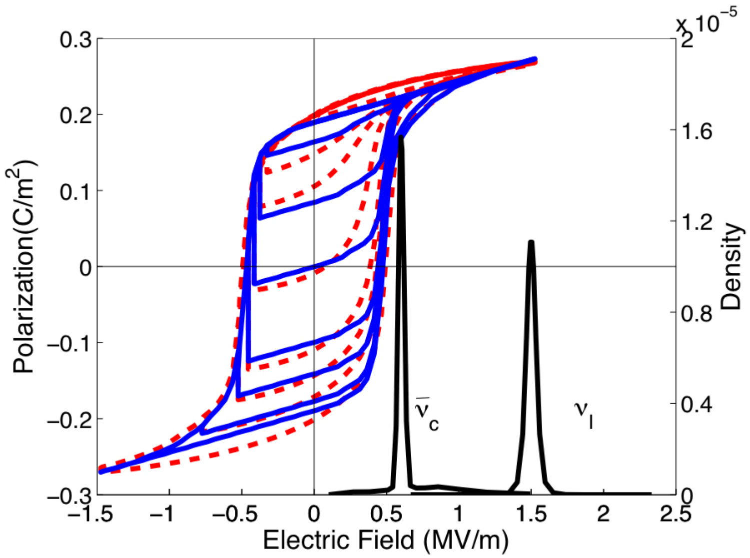


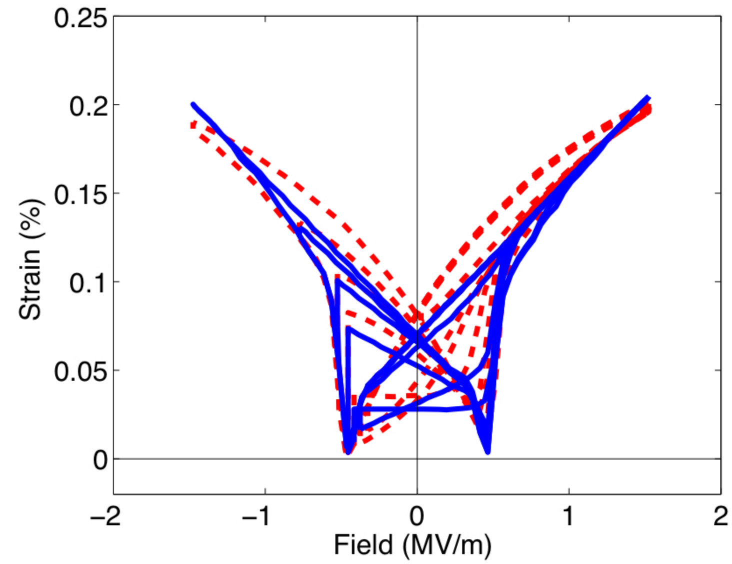
Homogenized Energy Model: Experimental Validation

PZT Data: York 2008



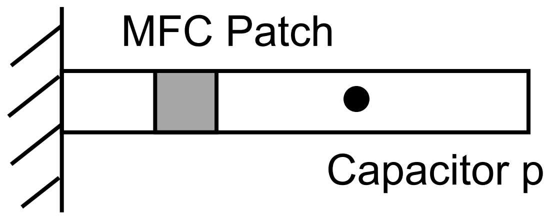






Structural Model: Macro-Fiber Composites (MFC)

Experimental Structure:



Beam Model:

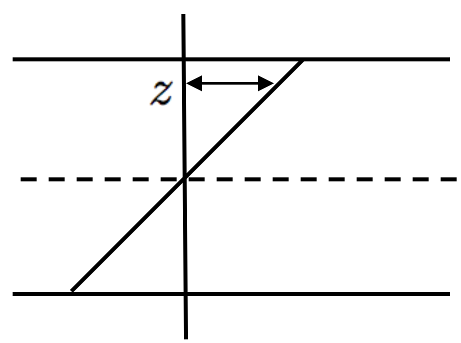
$$\rho\frac{\partial^2 w}{\partial t^2} - \frac{\partial^2 M}{\partial x^2} = f$$

Constitutive Relation: (Kelvin-Voigt damping)

$$\sigma(E,\varepsilon) = c^E\varepsilon + c_D\dot{\varepsilon} - e(E,\sigma_0)E - c^E\varepsilon_{irr}(E,\sigma_0)$$

Here

$$\varepsilon = \kappa z = -\frac{\partial^2 w}{\partial x^2}z$$



Moment:

$$M = \int_{\text{thickness}} \sigma z dz$$

$$\Rightarrow M = -c^E I\frac{\partial^2 w}{\partial x^2} - c_D I\frac{\partial^3 w}{\partial x^2 \partial t} - [k_1 e(E,\sigma_0)E + k_2\varepsilon_{irr}(E,\sigma_0)]\,\chi_{MFC}(x)$$

Uncertainty Quantification and Parameter Estimation

Sources of Uncertainty:

- Model
- Sensor measurements
- Initial/boundary conditions

Initial Strategy:

- Quantify uncertainty in parameters
- Propagate uncertainty through model

Parameters: $q = (q_{beam}, q_{hys})$

- Beam: $q_{beam} = (\rho, c^E I, C_D I, k_1, k_2)$
- HEM: $q_{hys} = (\varepsilon_R, \eta, \tau, \gamma, \sigma_c, \mu_c, \sigma_I, \alpha_i, \beta_j)$

Data-Driven Techniques:

- Used to obtain initial parameter estimates

Observation Process: Consider

$$w_j = w(t_j, \bar{x}; q) + \varepsilon_j$$

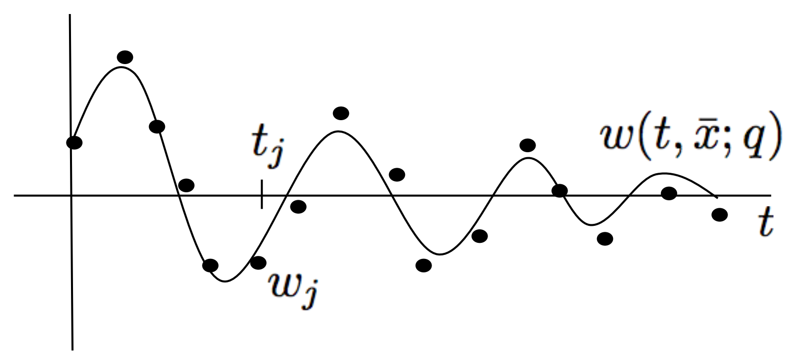
Data

Model Observations

IID Random Variable

$$E(\varepsilon_j) = 0$$

$$\text{var}(\varepsilon_j) = \sigma_0^2$$



Strategy: Treat q as random variable and determine covariance matrix or densities

Nonlinear Ordinary Least Squares

Parameter Values:

$$\hat{q} = \arg\min_{q \in Q} \sum_{j=1}^{N} [w_j - w(t_j, \bar{x}; q)]^2$$

Covariance Estimate:

$$\widehat{\mathrm{cov}(q)} = \hat{s}^2 \left[\chi^T(\hat{q})\chi(\hat{q})\right]^{-1}$$

Variance Estimate

Fisher Information Matrix

$$\chi_{jk}(\hat{q}) = \frac{\partial w(t_j, \bar{x}; \hat{q})}{\partial q_k}$$ Sensitivity Matrix

Problem:

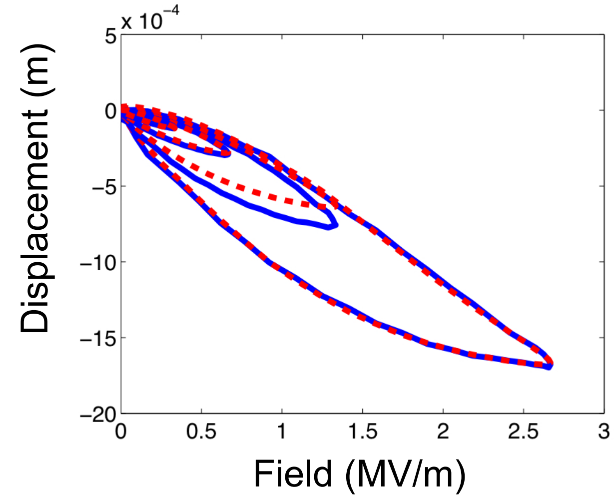
- Fisher information matrix ill-conditioned
- Redundant information

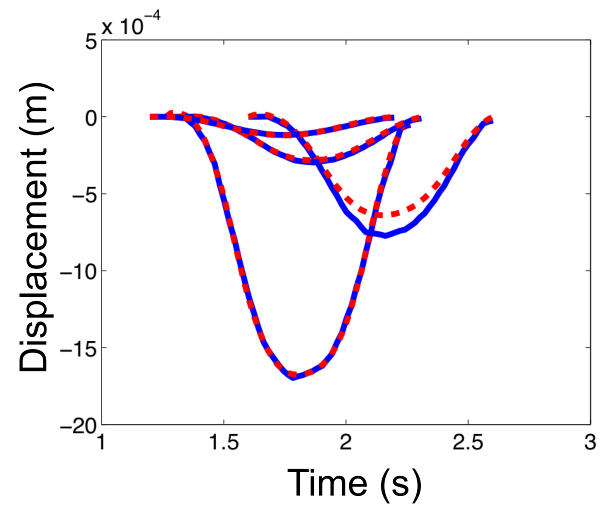
One Solution:

- Bootstrapping (resampling) techniques

MFC Values and Predictions:

η (m/A)	μ_c (MV/m)	σ_I (MV/m)	τ (s)
0.74×10^8	0.86	1.91	0.20×10^{-4}



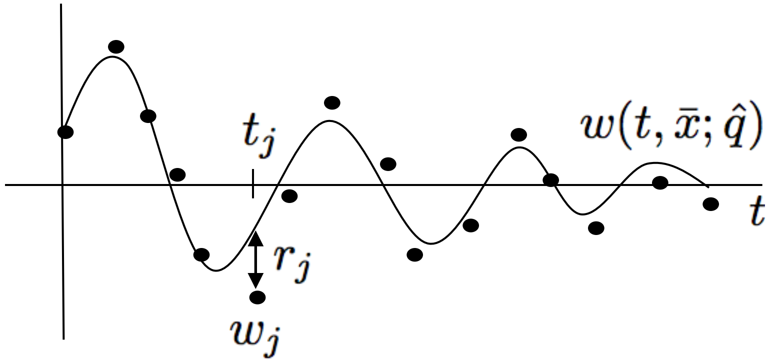


Residual Bootstrapping to Construct Parameter Densities

Algorithm:

1. Compute

$$\hat{q} = \arg\min_{q \in Q} \sum_{j=1}^{N} [w_j - w(t_j, \bar{x}; q)]^2$$



2. Compute residuals

$$r_j = w_j - w(t_j, \bar{x}; \hat{q}) \ , \ j = 1 \cdots , N$$

3. Compute bootstrapped data values

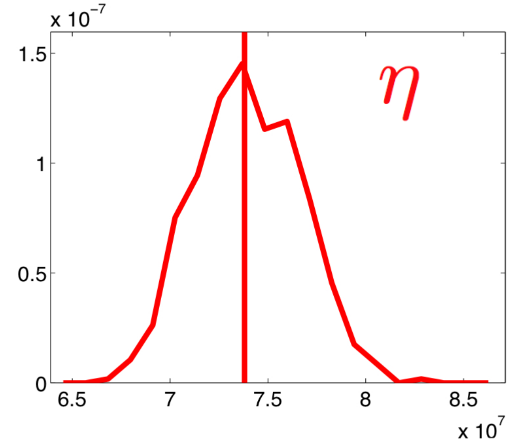
$$\hat{w}_j^k = w(t_j, \bar{x}; \hat{q}) + r_j v_j$$

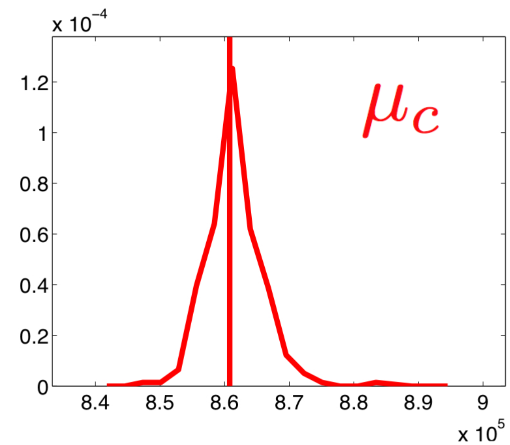
where v_j satisfies $E(v_j) = 0, E(v_j^2) = E(v_j^3) = 1$

4. Compute

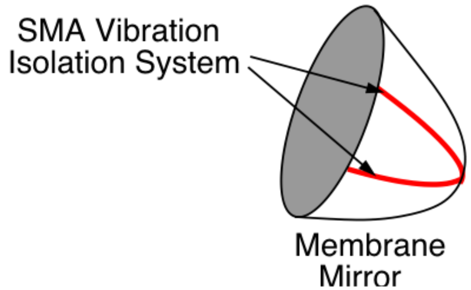
$$\hat{q} = \arg\min_{q \in Q} \sum_{j=1}^{N} [\hat{w}_j^k - w(t_j, \bar{x}; q)]^2$$

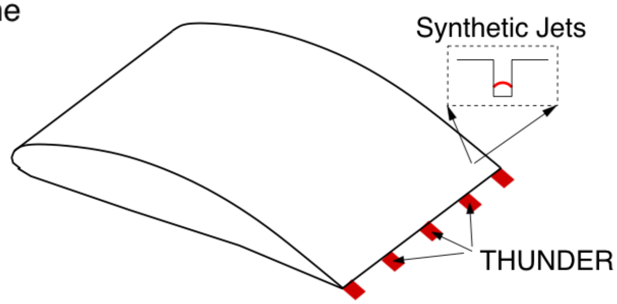
5. This yields K estimates of q

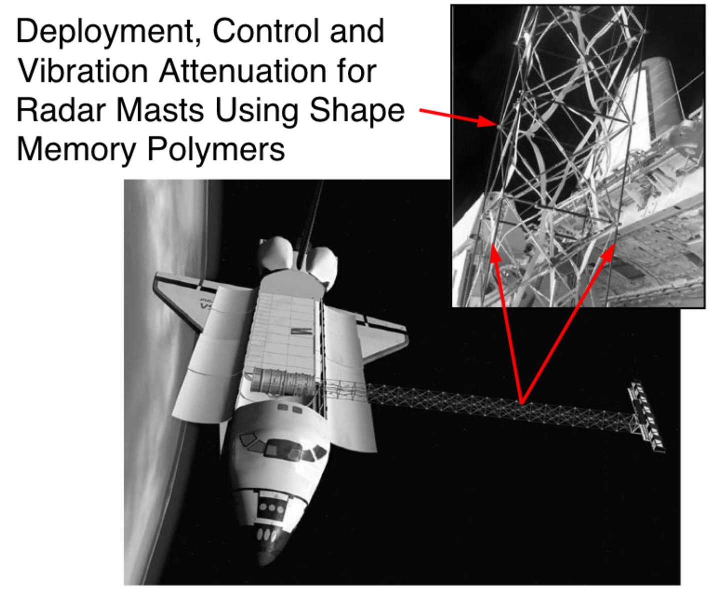




Model-Based Control Design







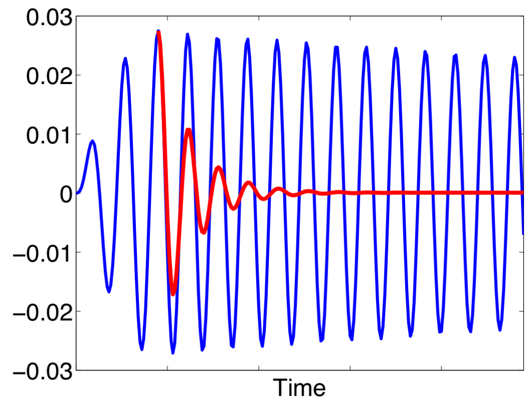
Simplistic Strategy:

Employ gains from linearized system

$$\frac{dy}{dt} = Ay(t) + Bu(t) + G(t)$$

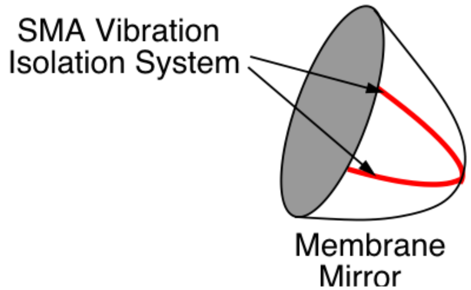
in nonlinear system

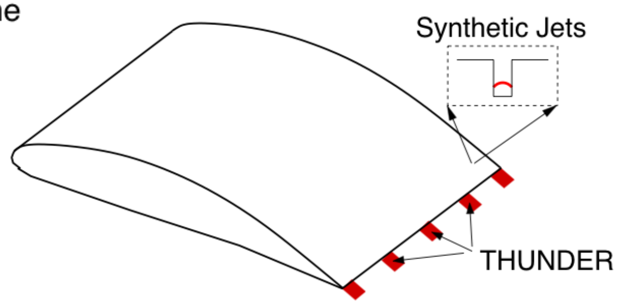
$$\frac{dy}{dt} = Ay(t) + [B(u,y)](t) + G(t)$$

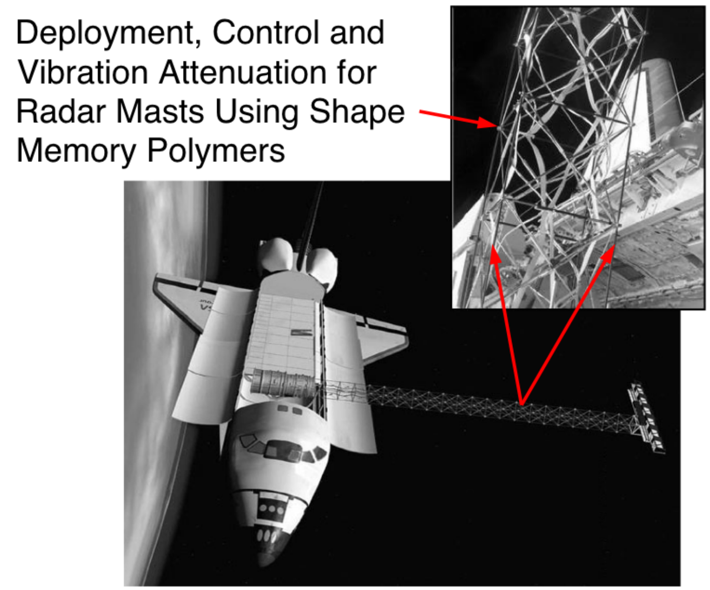


Low Input Level

Model-Based Control Design







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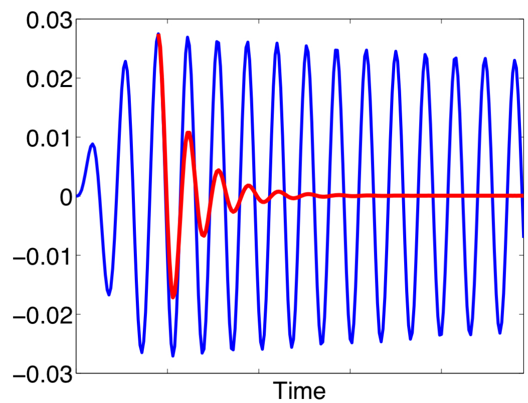
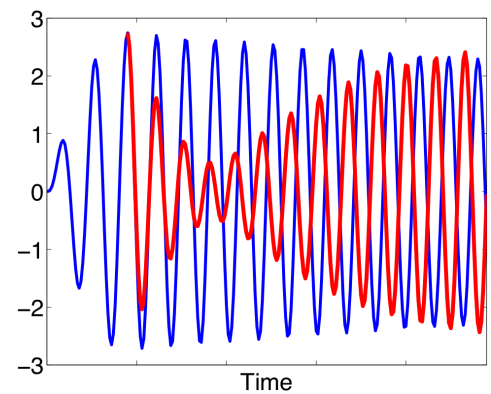


Low Input Level

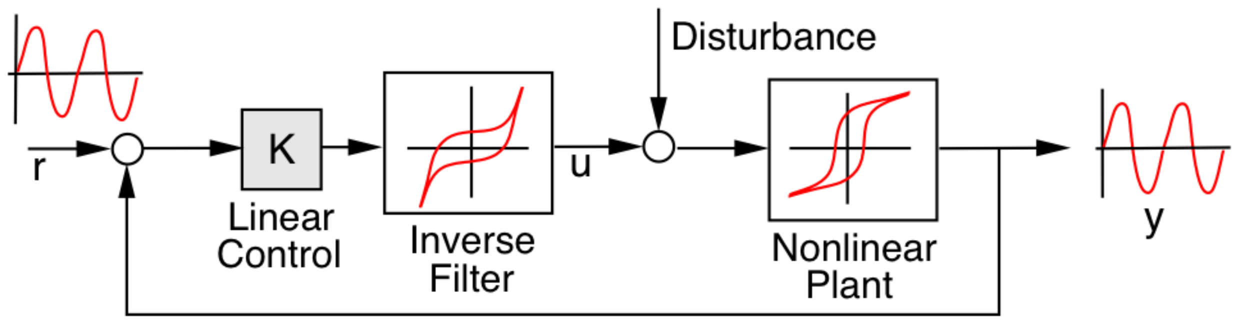


High Input Level

Nonlinear Model-Based Control Designs

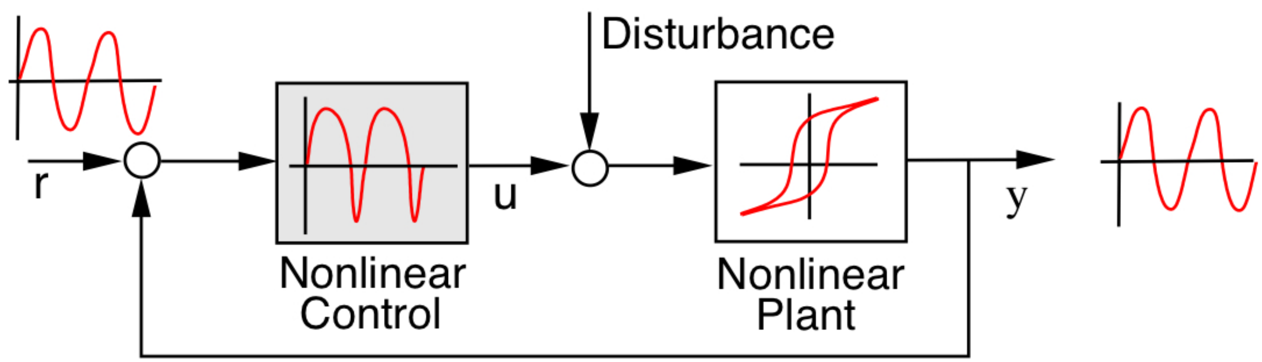
Nonlinear Inverse Filter/Linear Control:

- Employed by a number of researchers



Nonlinear Control:

- Synthesis between theory and experiments required for real-time implementation



Nonlinear Optimal Control

Function to be Minimized:

$$J(u) = \frac{1}{2} y^T(t_f) \Pi_f y(t_f) + \frac{1}{2} \int_0^{t_f} \left[y^T Q y + u^T R u \right] dt$$

Strategy: Form the Hamiltonian

$$H(y, \lambda, u) = \frac{1}{2} \left[y^T Q y + u^T R u \right] + \lambda^T \left[A y + B(u) + G \right]$$

Unconstrained optimization yields the necessary conditions

$$\dot{\lambda} = -\nabla_y H \quad \Rightarrow \quad \dot{\lambda}(t) = -A^T \lambda(t) - Q y(t)$$

$$0 = \nabla_u H \quad \Rightarrow \quad R u^*(t) + [B_u^T(u^*)](t)\, \lambda(t) = 0$$

Optimality System:

$$\begin{bmatrix} \dot{y}(t) \\ \lambda(t) \end{bmatrix} = \begin{bmatrix} A y(t) + [B(u)](t) + G(t) \\ -A^T \lambda(t) - Q y(t) \end{bmatrix} \quad , \quad \begin{array}{l} y(0) = y_0 \\ \lambda(t_f) = \Pi_f y(t_f) \end{array}$$

with $u^*(t) = -R^{-1} [B_u^T(u^*)](t)\, \lambda(t)$

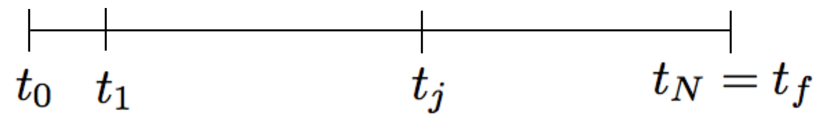
Numerical Method for Two-Point BVP

Optimality System: For $z = [y, \lambda]^T$, pose as

$$\dot{z}(t) = F(t, z)$$

$$E_0 z(0) = [y_0, 0]^T \quad , \quad E_T z(t_f) = [0, \Pi_f y(t_f)]^T$$

Solution Technique:



Discretize with forward difference and solve

$$\mathcal{F}(z_h) = 0$$

using the quasi-Newton iteration

$$z_h^{m+1} = z_h^m + \xi_h^m \quad \text{where} \quad \mathcal{F}'(z_h^m)\,\xi_h^m = -\mathcal{F}(z_h^m)$$

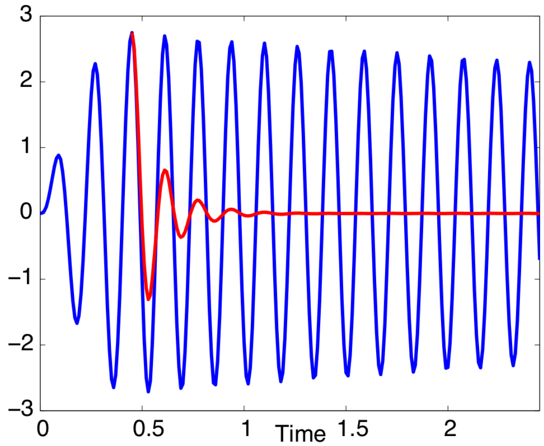
Note:

$$\mathcal{F}'(z_h^m) = \begin{bmatrix} S_1 & R_1 & & & \\ & S_2 & R_2 & & \\ & & \ddots & \ddots & \\ & & & S_N & R_N \\ E_0 & & & & E_T \end{bmatrix}$$

- Employ analytic LU decomposition
- 2-D examples: have run over 500,000 unknowns
- Open loop computation for later experimental example: ~7 seconds

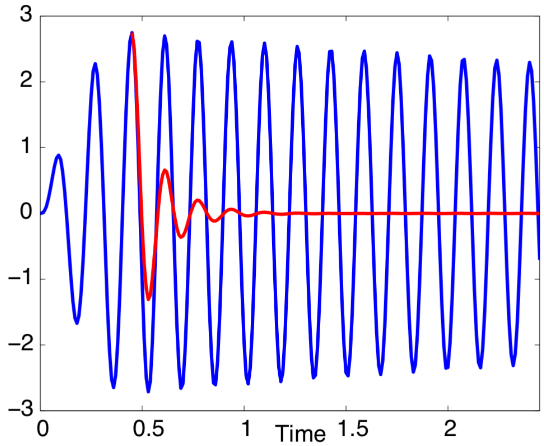
Nonlinear Control -- Open Loop

Nonlinear Control

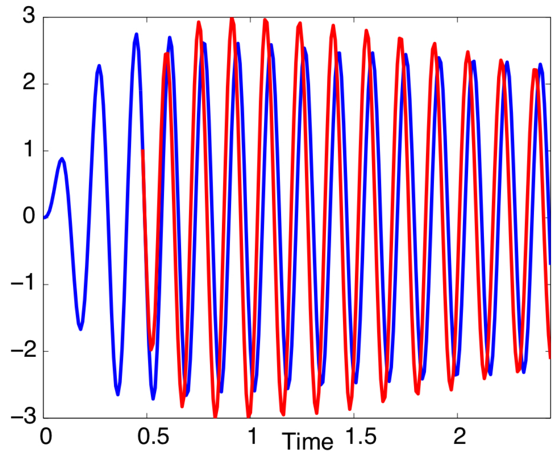


Nonlinear Control -- Open Loop with Delay

Nonlinear Control



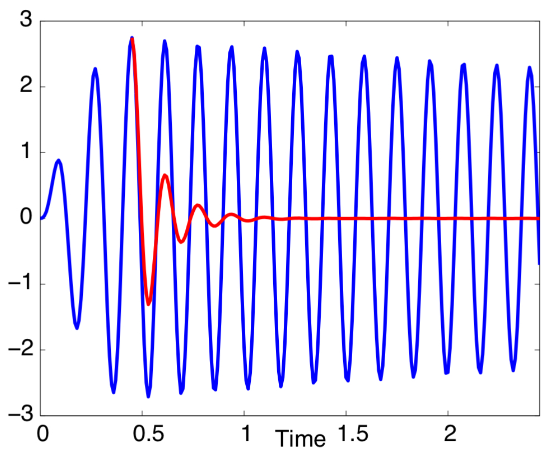
0.03 Second Delay



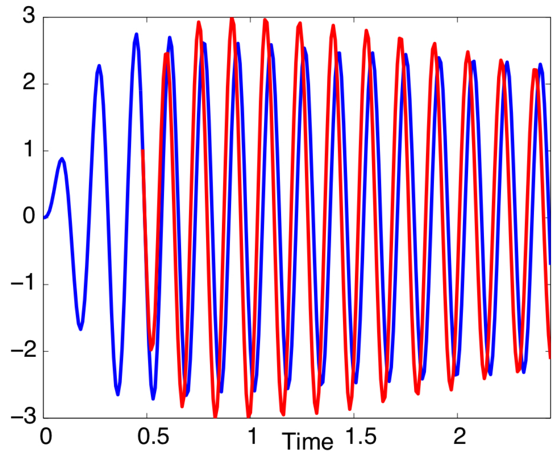
Problem: Open loop control not robust; e.g., 0.03 second delay

Nonlinear Control -- Perturbation Feedback

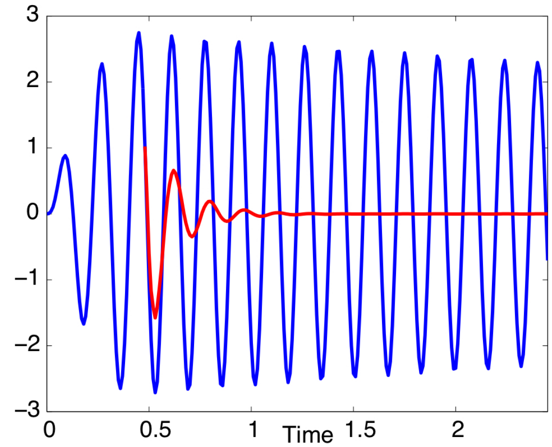
Nonlinear Control



0.03 Second Delay

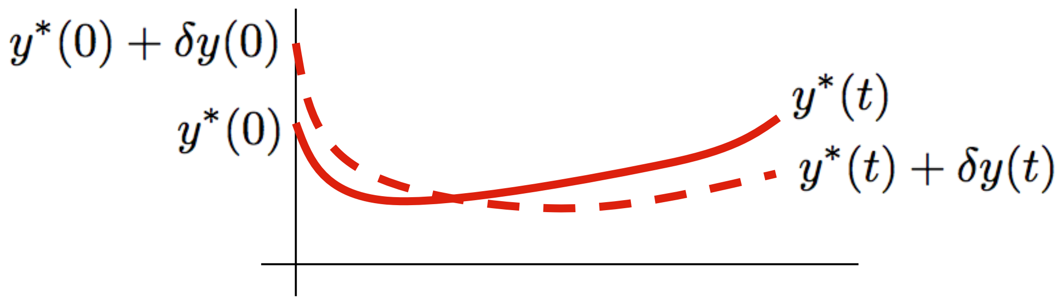


Perturbation Feedback



Problem: Open loop control not robust; e.g., 0.03 second delay

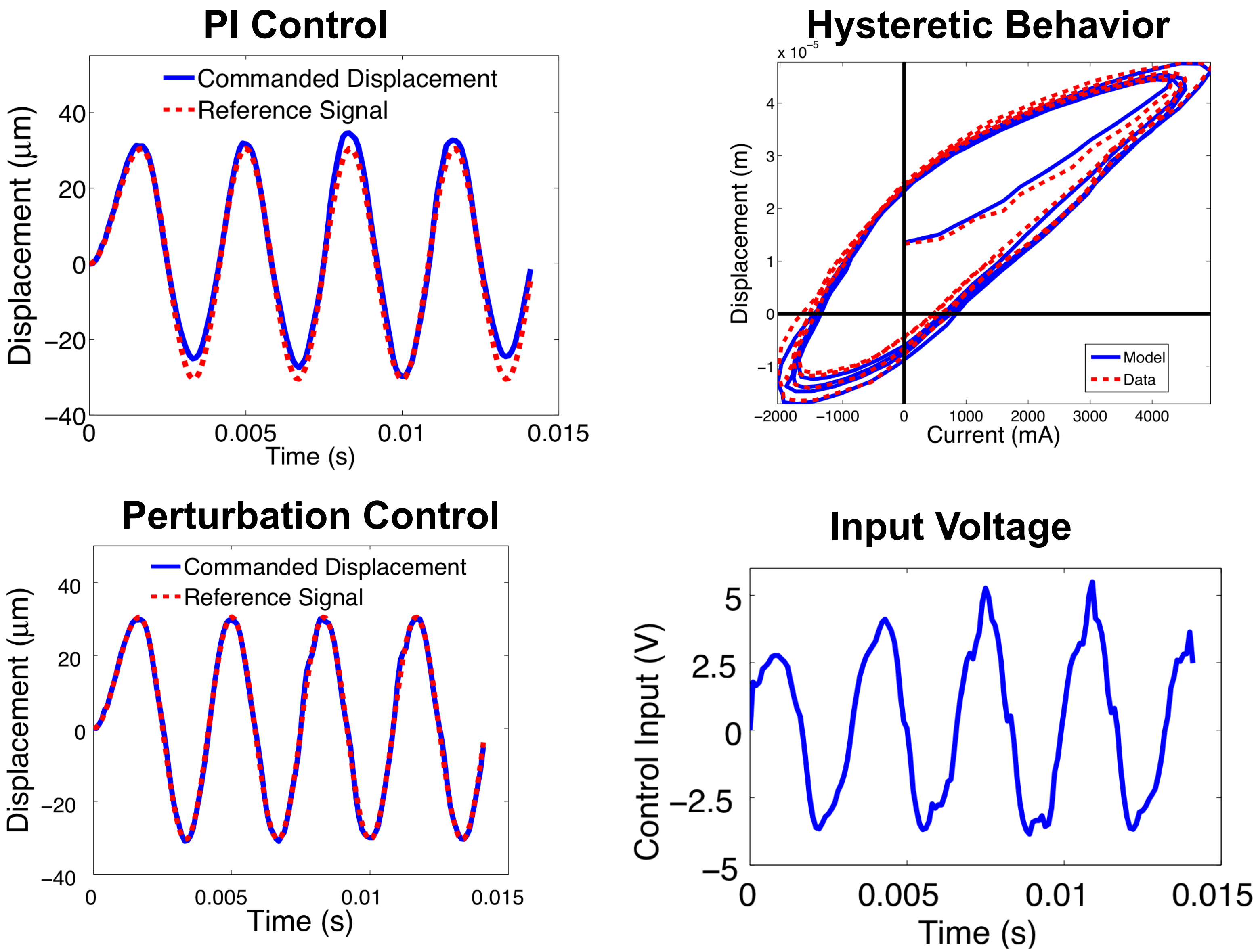
Strategy: Feedback around optimal trajectory $(u^*(t), y^*(t))$



PI Perturbation Control: $\delta u(t) = -k_I e(t) - k_I \int_0^t e(s) ds$

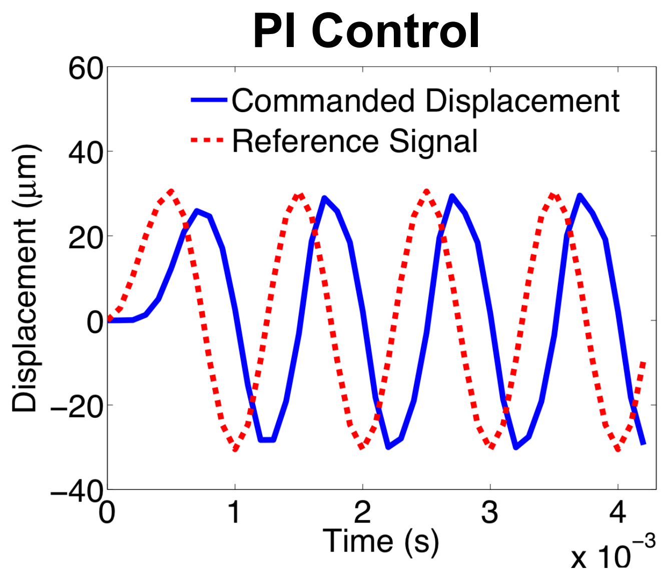
Narrowband Optimal Control:

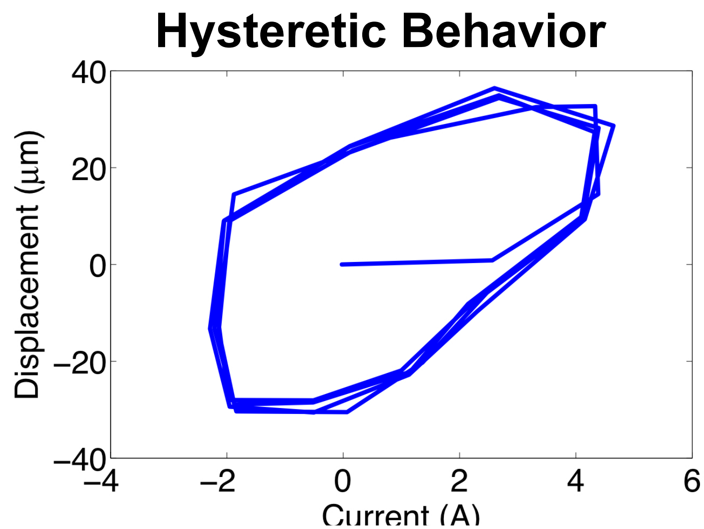
Experimental Implementation --- Tracking at 300 Hz

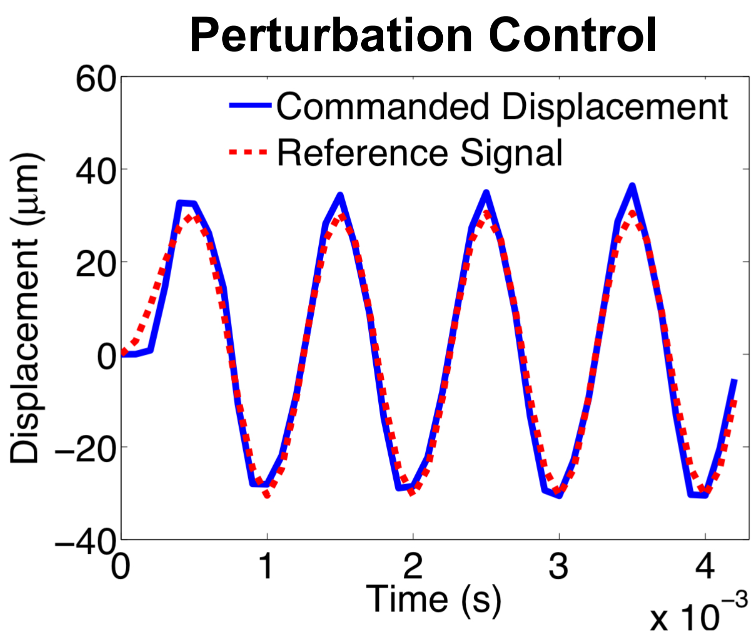


Observation: PI starts to break down at 300 Hz

Experimental Implementation --- Tracking at 1000 Hz







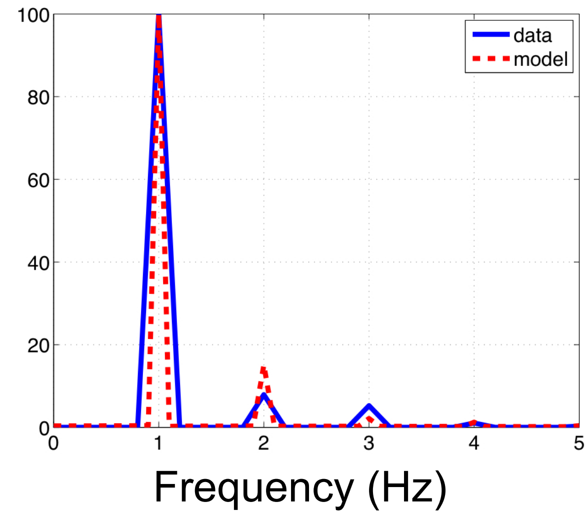
Observation:

- Model fit at 300 Hz and 500 Hz

--- it is predicting at 1000 Hz

Narrowband Perturbation Feedback

Recall: Hysteresis nonlinearity can produce higher harmonics



Filter Equations:

$$\frac{dx_f}{dt} = A_f x_f(t) + BCx(t)$$

$$A_{fi} = \begin{bmatrix} -2\xi_i\omega_i & -\omega_i^2 \\ 1 & 0 \end{bmatrix}$$

Note: ω_i is a frequency being targeted

ξ_i is an associated damping coefficient

Control Law:

$$u(t) = u^*(t) + u_{NB}(t) + u_I(t)$$

Optimal Control ($u^*(t)$)

Narrowband Feedback ($u_{NB}(t)$)

Integral Feedback ($u_I(t)$)

$$u_{NB} = -[K_f \quad K][x_f \ ; \ e]$$

Narrowband Perturbation Feedback --- Experimental Results

Recall: Hysteresis nonlinearity can produce higher harmonics

Filter Equations:

$$\frac{dx_f}{dt} = A_f x_f(t) + BCx(t)$$

$$A_{fi} = \begin{bmatrix} -2\xi_i\omega_i & -\omega_i^2 \\ 1 & 0 \end{bmatrix}$$

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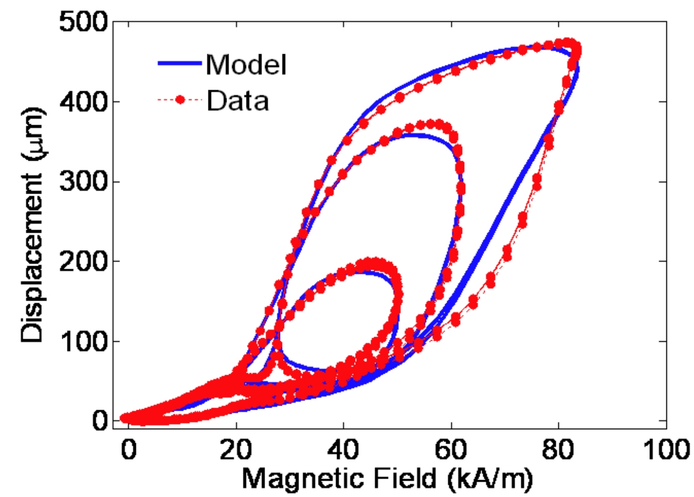
Control Law:

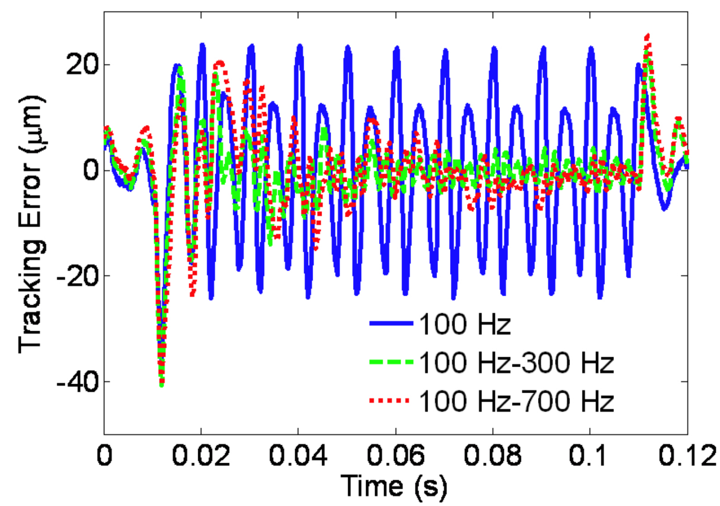
$$u(t) = u^*(t) + u_{NB}(t) + u_I(t)$$

Optimal Control ($u^*(t)$); Narrowband Feedback ($u_{NB}(t)$); Integral Feedback ($u_I(t)$)

$$u_{NB} = -[K_f \quad K][x_f \; ; \; e]$$

Note: 450 μm Max Displacements





Concluding Remarks

Material Properties:

- Hysteresis and constitutive nonlinear inherent to high performance smart materials.
- Hysteresis and nonlinearities can be advantageous

Nonlinear Model Development:

- Physics-based models suitably accurate and efficient for design and control applications.

Uncertainty Quantification:

- Bootstrapping permits characterization of non-Gaussian parameter densities.
- Monte Carlo/bootstrapping methods used to construct confidence bounds for model since not limited by number of parameters.

Control Design:

- Perturbation designs permit real-time implementation.