Boundary control of infinite-dimensional port-Hamiltonian systems with dissipation using invariant function approach.

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Infinite dimensional port Hamiltonian systems :

- Material and energy balance equations $\longrightarrow$ physically consistent model.
- Definition of the geometric structure (Dirac structure) and of the boundary port variables $\longrightarrow$ derivation of boundary control systems.
- The core of the approach is the energy of the system and its links with the dynamics and the environment.

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New issue for system control theory

Modelling step is important $\rightarrow$ the physical properties can be advantageously used for analysis, simulation and control purposes

In this talk :

- Boundary control of infinite dimensional system using the energy shaping approach and the immersion/reduction method.
  - Controller under port Hamiltonian format.
  - Power preserving interconnection.
  - Use of Casimir invariant (to link controller states to system states).
- Casimir functions :
  - In the power preserving case : dynamical and structural invariants obtained from Poisson Bracket.
  - In the case of system with dissipation : structural invariants obtained from Leibnitz Bracket. Not necessarily dynamical invariants
- Chosen illustrative example :

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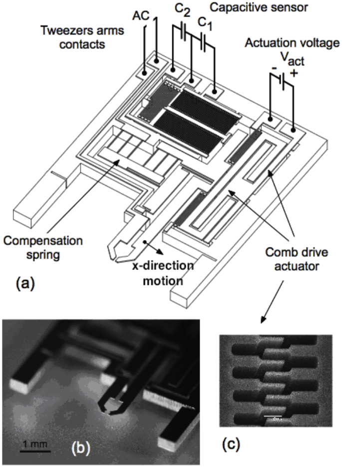
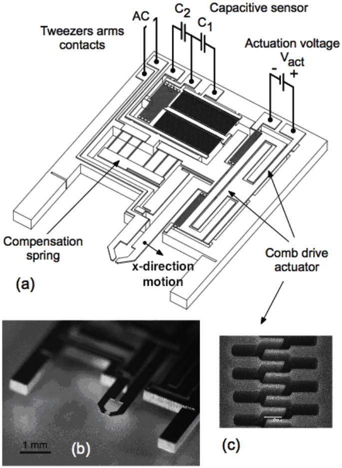
Control of microsystems : Nanotweezers for DNA manipulation

Considered system

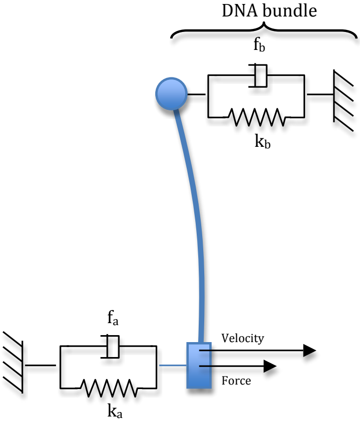


- Objective : nano manipulation and DNA characterization



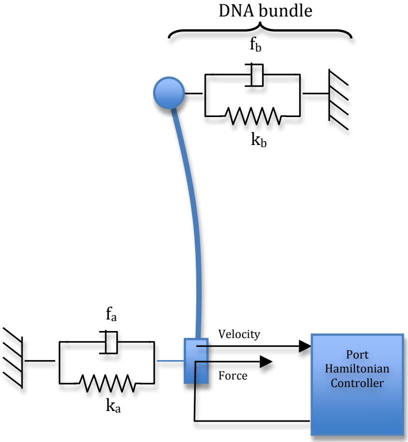
- Actuator : electrostatic comb drive → force proportional to the square of the applied voltage $F_c = f(V^2)$
- Sensor : electrostatic comb drive+capacitor → velocity

Model



- Mass spring damper + Timoshenko beam + mass spring damper

Model



- Mass spring damper + Timoshenko beam + mass spring damper
- Port Hamiltonian controller

- Beam model :
    - State (energy) variables ($w(z,t)$ is the transverse displacement and $\phi(z,t)$ the rotation angle) :

$$x = \begin{bmatrix} \frac{\partial w}{\partial z} - \phi \\ \rho \frac{\partial w}{\partial t} \\ \frac{\partial \phi}{\partial z} \\ I_\rho \frac{\partial \phi}{\partial t} \end{bmatrix} \begin{matrix} \longrightarrow \text{shear displacement} \\ \longrightarrow \text{transverse momentum distribution,} \\ \longrightarrow \text{angular displacement,} \\ \longrightarrow \text{angular momentum distribution.} \end{matrix}$$

    - Effort variables and energy :

$$e = \begin{bmatrix} F \\ v \\ T \\ \omega \end{bmatrix} \begin{matrix} \longrightarrow \text{longitudinal force,} \\ \longrightarrow \text{velocity,} \\ \longrightarrow \text{torque,} \\ \longrightarrow \text{angular velocity.} \end{matrix} \quad ; \; \mathcal{H}_{bm} = \frac{1}{2} \int_0^L \left(K x_1^2 + \frac{x_2^2}{\rho} + EI x_3^2 + \frac{x_4^2}{I_\rho} \right) dz$$

- DNA
- Combdrive+suspension system

- Beam model :
  - From balance equations :

$$\underbrace{\frac{\partial x}{\partial t}}_{f} = \underbrace{\begin{bmatrix} 0 & \frac{\partial}{\partial z} & 0 & -1 \\ \frac{\partial}{\partial z} & -f_{bm} & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial}{\partial z} \\ 1 & 0 & \frac{\partial}{\partial z} & 0 \end{bmatrix}}_{\mathcal{J}_{bm}-\mathcal{R}_{bm}} \underbrace{\begin{bmatrix} F \\ v \\ T \\ \omega \end{bmatrix}}_{e}$$

  That can be written :

$$\frac{\partial x}{\partial t} = \left(P_1 \frac{\partial}{\partial z} + P_0 + G_0 \right) \mathcal{L} x \text{ with } P_1 = P_1^T,\ P_0 = -P_0^T,\ G_0 = G_0^T$$

- DNA
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- Beam model :
  - From balance equations :

$$\underbrace{\frac{\partial x}{\partial t}}_{f} = \underbrace{\begin{bmatrix} 0 & \frac{\partial}{\partial z} & 0 & -1 \\ \frac{\partial}{\partial z} & -f_{bm} & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial}{\partial z} \\ 1 & 0 & \frac{\partial}{\partial z} & 0 \end{bmatrix}}_{\mathcal{J}_{bm}-\mathcal{R}_{bm}} \underbrace{\begin{bmatrix} F \\ v \\ T \\ \omega \end{bmatrix}}_{e}$$

Considering $f_{bm} = 0$ one can choose as boundary port variables as :

$$\begin{bmatrix} f_\partial \\ e_\partial \end{bmatrix} = U \begin{bmatrix} P_1 & -P_1 \\ I & I \end{bmatrix} \begin{bmatrix} \mathcal{L}x(b) \\ \mathcal{L}x(a) \end{bmatrix} \text{ with } U^T \Sigma U = \Sigma$$

- DNA
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- Beam model :
  - From balance equations :

$$\underbrace{\frac{\partial x}{\partial t}}_{f} = \underbrace{\begin{bmatrix} 0 & \frac{\partial}{\partial z} & 0 & -1 \\ \frac{\partial}{\partial z} & -f_{bm} & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial}{\partial z} \\ 1 & 0 & \frac{\partial}{\partial z} & 0 \end{bmatrix}}_{\mathcal{J}_{bm}-\mathcal{R}_{bm}} \underbrace{\begin{bmatrix} F \\ v \\ T \\ \omega \end{bmatrix}}_{e}$$

A possible choice is :

$$\begin{bmatrix} f_\partial \\ e_\partial \end{bmatrix} = \begin{bmatrix} v(b) \\ \omega(b) \\ -v(a) \\ -\omega(a) \\ F(b) \\ T(b) \\ F(a) \\ T(a) \end{bmatrix}, \text{ and } u = \begin{bmatrix} v(b) \\ \omega(b) \\ -v(a) \\ -\omega(a) \end{bmatrix} y = \begin{bmatrix} F(b) \\ T(b) \\ F(a) \\ T(a) \end{bmatrix}$$

- DNA
- Combdrive+suspension system

- Beam model :
- DNA

  From balance equations ($q_b = \begin{bmatrix} w_b \\ \phi_b \end{bmatrix}$ gen. coord.,
  $p_b = \begin{bmatrix} M_b \frac{dw_b}{dt} \\ J \frac{d\phi_b}{dt} \end{bmatrix}$ gen. moment.,
  $\mathcal{H}(q_b, p_b) = \frac{1}{2}\left(\frac{p_{b1}^2}{M_b} + \frac{p_{b2}^2}{J_b} + k_b q_{b1}^2\right))$:

$$\begin{cases} \frac{d}{dt}\begin{bmatrix} q_b \\ p_b \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & I \\ -I & -D_b \end{bmatrix}}_{J_b - R_b} \begin{bmatrix} \partial_{q_b}\mathcal{H}(q_b, p_b) \\ \partial_{p_b}\mathcal{H}(q_b, p_b) \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \begin{bmatrix} F_b \\ T_b \end{bmatrix} \\ \begin{bmatrix} v_b \\ \omega_b \end{bmatrix} = \begin{bmatrix} 0 & I \end{bmatrix} \begin{bmatrix} \partial_{q_b}\mathcal{H}(q_b, p_b) \\ \partial_{p_b}\mathcal{H}(q_b, p_b) \end{bmatrix} \end{cases}$$

- Combdrive+suspension system

- Beam model :
- DNA
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  From balance equations ($q_a = \begin{bmatrix} w_a \\ \phi_a \end{bmatrix}$ gen. coord.,

  $p_a = \begin{bmatrix} M_a \frac{dw_a}{dt} \\ J\frac{d\phi_a}{dt} \end{bmatrix}$ gen. moment.,

  $\mathcal{H}(q_a, p_a) = \frac{1}{2}\left(\frac{p_{a1}^2}{M_b} + \frac{p_{a2}^2}{J_a} + k_a q_{a1}^2\right))$:

$$\begin{cases} \frac{d}{dt}\begin{bmatrix} q_a \\ p_a \end{bmatrix} = \begin{bmatrix} 0 & I \\ -I & -D_a \end{bmatrix} \begin{bmatrix} \partial_{q_a}\mathcal{H}(q_a,p_a) \\ \partial_{p_a}\mathcal{H}(q_a,p_a) \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \begin{bmatrix} F_a \\ T_a \end{bmatrix} \\ \begin{bmatrix} v_a \\ \omega_a \end{bmatrix} = \begin{bmatrix} 0 & I \end{bmatrix} \begin{bmatrix} \partial_{q_a}\mathcal{H}(q_a,p_a) \\ \partial_{p_a}\mathcal{H}(q_a,p_a) \end{bmatrix} \end{cases}$$

- Beam model :
- DNA
- Combdrive+suspension system +controller
  Controller :

$$\left\{ \begin{array}{rcll} \frac{d}{dt}x_c & = & (J_c - R_c) & \partial_{x_c}\mathcal{H}(x_c) + G_c \begin{bmatrix} v_a \\ \omega_a \end{bmatrix} \\ \begin{bmatrix} F_c \\ T_c \end{bmatrix} & = & G_c^T & \partial_{x_c}\mathcal{H}(x_c) \end{array} \right.$$

- Beam model :
- DNA
- Combdrive+suspension system +controller
  From balance equations :

$$
\left\{
\begin{array}{lcl}
\frac{d}{dt}\begin{bmatrix} q_a \\ p_a \\ x_c \end{bmatrix} & = & \begin{bmatrix} 0 & I & 0 \\ -I & -D_a & -G_c^T \\ 0 & G_c & J_c - R_c \end{bmatrix} \begin{bmatrix} \partial_{q_a}\mathcal{H}(q_a, p_a, x_c) \\ \partial_{p_a}\mathcal{H}(q_a, p_a, x_c) \\ \partial_{x_c}\mathcal{H}(q_a, p_a, x_c) \end{bmatrix} + \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix} \left[\begin{array}{c} F_a \\ T_a \end{array} \right. \\
\begin{bmatrix} v_a \\ \omega_a \end{bmatrix} & = & \begin{bmatrix} 0 & I & 0 \end{bmatrix} \begin{bmatrix} \partial_{q_a}\mathcal{H}(q_a, p_a, x_c) \\ \partial_{p_a}\mathcal{H}(q_a, p_a, x_c) \\ \partial_{x_c}\mathcal{H}(q_a, p_a, x_c) \end{bmatrix}
\end{array}
\right.
$$

Power preserving interconnexion :

$$\begin{cases} u_{beam} = \begin{bmatrix} u_{beam,a} \\ u_{beam,b} \end{bmatrix} = \begin{bmatrix} f_{\partial,a} \\ f_{\partial,b} \end{bmatrix} = \begin{bmatrix} -y_a \\ -y_b \end{bmatrix} \\ u = \begin{bmatrix} u_a \\ u_b \end{bmatrix} = \begin{bmatrix} e_{\partial,a} \\ e_{\partial,b} \end{bmatrix} \end{cases}$$

The closed loop operator $f = (\mathcal{J}_t - \mathcal{R}_t)\, e$ is equal to :

$$\mathcal{J}_t - \mathcal{R}_t = \left[\begin{array}{c|ccc|cc} \mathcal{J}_{bm} - \mathcal{R}_{bm} & 0 & 0 & 0 & \multicolumn{2}{c}{0} \\ \hline 0 & 0 & I & 0 & & \\ G_a & -I & -D_a & -G_c^T & \multicolumn{2}{c}{0} \\ 0 & 0 & G_c & J_c - R_c & & \\ \hline G_b & & & & 0 & I \\ 0 & & 0 & & -I & -D_b \end{array}\right]$$

$$G_{a,b} = \begin{bmatrix} 1|_{a,b} & 0 & 0 & 0 \\ 0 & 0 & 1|_{a,b} & 0 \end{bmatrix} \text{ and } \mathcal{D}(\mathcal{J}_t - \mathcal{R}_t) = \{ e \in H^1 | e_{p_{a,b}} = -G_{a,b} e \}$$

- In the conservative case $f = \mathcal{J}e$:
  - Space of *admissible efforts* :

$$\mathcal{E}_{\text{adm}} = \{e \in \mathcal{E} | \exists f \in \mathcal{F} \text{ such that } (f, e) \in \mathcal{D}\}$$

  - Skew symmetric bilinear form on $\mathcal{E}_{\text{adm}}$

$$[e_1, e_2] := < e_1 | f_2 > \in L, \quad f_2 \in \mathcal{F} \text{ such that } (f_2, e_2) \in \mathcal{D}$$

  - Set of *admissible functions*

$$\begin{aligned} K_{\text{adm}} &= \{k : \mathcal{F} \to \mathbb{R} | \forall a \in \mathcal{F} \ \exists e \in \mathcal{E}_{\text{adm}} \text{ such that } \forall \delta a \in \mathcal{F}, \\ & \qquad \forall \eta \in \mathbb{R}, k(a + \eta \delta a) = k(a) + \eta < e | \delta a > + o(\eta)\} \end{aligned}$$

e is the *derivative* of k at a, is denoted by $\delta k(a)$

- In the conservative case $f = \mathcal{J} e$: on K_{adm} we define

$$\{k_1, k_2\}(a) := [\delta k_1(a), \delta k_2(a)], \quad k_1, k_2 \in K_{\text{adm}}$$

$\{,\}$ defines a *pseudo-Poisson bracket*.

  - By skew-symmetry of $[\ ,\ ]$ it immediately follows that also $\{,\}$ is *skew-symmetric*
  - Satisfies the Jacobi identity (in the linear case)$\{x, \{y, z\}\} + \{z, \{x, y\}\} + \{y, \{z, x\}\} = 0$

Hamiltonian system are defined by : $\dot{x} = \{x, H(x)\}$

The Casimir functions are the functions $C \in K_{adm}$ such that :

$$\{k, C\} = [\delta k, \delta C] = 0, \ \ \forall k \in K_{adm}$$

In this case :
$\frac{dC}{dt} = \frac{\partial C}{\partial x}^T \frac{\partial x}{\partial t} = [\delta C, \delta H] = \{C, H\} = -\{H, C\} = 0$

- In the dissipative case $f = (\mathcal{J} - \mathcal{R})\, e$: we consider $f_0 = \mathcal{J}\, e$
  - Space of *admissible efforts* :

$$\mathcal{E}_{\text{adm}} = \{e \in \mathcal{E} | \exists f_0 \in \mathcal{F} \text{ such that } (f_0, e) \in \mathcal{D}_{\mathcal{J}}\}$$

  - Bilinear form on $\mathcal{E}_{\text{adm}}$

$[e_1, e_2] := \langle e_1 | f_0 \rangle - \langle e_1 | \mathcal{R} e_2 \rangle \in L, \quad f_0 \in \mathcal{F} \;$ such that $\; (f_0, e_1) \in \mathcal{D}_{\mathcal{J}}$

on K_{adm} we define

$$\{k_1, k_2\}(a) := [\delta k_1(a), \delta k_2(a)], \quad k_1, k_2 \in K_{\text{adm}}$$

$\{,\}$ defines a *Leibnitz bracket*.Dissipative port Hamiltonian system are defined by : $\dot{x} = \{x, H(x)\}$

The right Casimir functions are the functions $C \in K_{adm}$ such that :

$$\{k, C\} = [\delta k, \delta C] = 0, \quad \forall k \in K_{adm}$$

In this case : $\frac{dC}{dt} = \frac{\partial C}{\partial x}^T \frac{\partial x}{\partial t} = \{C, H\} \neq -\{H, C\} \nRightarrow \frac{dC}{dt} = 0$

Idea

From the closed loop system dynamics :

$$\frac{d}{dt}\begin{bmatrix} x \\ x_c \end{bmatrix} = (\mathcal{J}_{tot} - \mathcal{R}_{tot})\begin{bmatrix} \delta_x H_{cl}(x, x_c) \\ \delta_{x_c} H_{cl}(x, x_c) \end{bmatrix}$$

shape the closed loop energy function :

$$H_{cl}(x, x_c) = H(x) + H_c(x_c)$$

by restricting the controller dynamics using Casimir invariants of the form :

$$\mathcal{C} = x_c + F(x)$$

Then

$$H_{cl}(x, x_c) = H(x) + H_c(\mathcal{C} - F(x))$$

It remains to choose H_c such that : $\delta H_{cl}(x^*) = 0$+stability

Back to the example : the right Casimir invariants are defined such that :

$$\{k, C\} = [\delta k, \delta C] = 0 \;\; \forall k, C \in K_{adm}$$

i.e. for $\delta_x C \in \mathcal{D}(\mathcal{J}_t - \mathcal{R}_t)$

$$\begin{cases} (\mathcal{J}_{bm} - \mathcal{R}_{bm})\,\delta_x \mathcal{C} = 0 \\ \delta_{p_a}\mathcal{C} = 0 \\ G_a \delta_x \mathcal{C} - \delta_{q_a}\mathcal{C} - D_a \delta_{p_a}\mathcal{C} - G_c^T \delta_{x_c}\mathcal{C} = 0 \\ G_c \delta_{p_a}\mathcal{C} + (J_c - R_c)\,\delta_{x_c}\mathcal{C} \\ \delta_{p_b}\mathcal{C} = 0 \\ G_b \delta_x \mathcal{C} - \delta_{q_b}\mathcal{C} - D_b \delta_{p_b}\mathcal{C} = 0 \end{cases}$$

Choosing $J_c = R_c = 0, \;\; G_c = I$ and :

$$\mathcal{C}_i(x, q_a, p_a, q_b, p_b, x_c) = x_{ci} + F_i(x, p_a, q_b, p_b)$$

one can find :

$$\mathcal{C}_1 = x_{c1} - q_{1,a} + 2q_{1,b} - 2Lq_{2,b} - 2\int_0^L (x_1 + zx_3)\,dz, \;\; \mathcal{C}_2 = x_{c2} + q_{2,a} + 2q_{2,b} + 2\int_0^L x_3\,dz$$

One can express the controller state from the system state by :

$$x_{c1} = q_{1,a} - 2Lq_{2,b} + 2\int_0^L (x_1 + zx_3)\,dz - 2q_{1,b} + C_1, \quad x_{c2} = q_{2,a} - 2q_{2,b} - 2\int_0^L x_3\,dz + C_2$$

It remains to choose the controller Hamiltonian function in order to shape the closed loop energy function. The desired equilibrium is given by :

$$F(L) = -k_b x^*, T^*(L) = 0, v^*(L) = \omega^*(L) = 0, v(0) = \omega^*(L) = 0, \phi^* = 0$$

That leads to :

$$\phi^* = \frac{mg}{2EI}\left[(z-L)^2 - L^2\right]$$

$$w^* = \frac{mg}{2EI}(z-L)^3 - \left(\frac{mgL^2}{2EI} + \frac{mg}{K}\right)(z-L) - k_b x^*$$

$$\Longrightarrow \Xi^* = (x_1^*, x_2^*, x_3^*, x_4^*, q_a^*, p_a^*, x_c^*, q_b^*, p_b^*)$$

$$\mathcal{H}_{cl} = \frac{1}{2}\int_0^L \left(Kx_1^2 + \frac{x_2^2}{\rho} + EIx_3^2 + \frac{x_4^2}{I_\rho}\right) dz$$
$$+\frac{1}{2}\left(\frac{p_{a1}^2}{M_a} + \frac{p_{a2}^2}{J_a} + k_a q_{a1}^2\right) + \frac{1}{2}\left(\frac{p_{b1}^2}{M_b} + \frac{p_{b2}^2}{J_b} + k_b q_{b1}^2\right) + H_c(x_{c1}, x_{c2})$$

Search of admissible Lyapunov function through H_c

i.e.

$$H_c(x_{c1}, x_{c2}) = H_c(q_a, x_1, x_3, q_b)$$

such that $\mathcal{H}_{cl}$ has a minimum in Ξ^* :

- $\partial_\Xi \mathcal{H}_{cl}(\Xi^*) = 0$
- there exist $\gamma, \Gamma_1, \Gamma_2 > 0$ such that $\Gamma_1\|\delta\Xi\| \leq \mathcal{H}_{cl}(\Xi^* + \delta\Xi) - \mathcal{H}_{cl}(\Xi^*) \leq \Gamma_2\|\delta\Xi\|^\gamma$

Ex : $H_c(x_{c1}, x_{c2}) =$
$-K_1(x_{c1} - x_{c1}^*)^2 - K_2(x_{c2} - x_{c2}^*)^2 + M_1(x_{c1} - x_{c1}^*) + M_2(x_{c2} - x_{c2}^*)$

What has been done :

- Definition of the right Casimir invariant derived from Leibnitz bracket.
- Use of the right Casimir invariant derived from Leibnitz bracket for control purpose.
- A first application to nanotweezers.

Ongoing researches :

- Proof of stability for a class of controllers using results obtained for PHS.
- Application to dissipative differential operators.
- Other controller parametrizations.

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