Use of mobile actuator and sensor network with augmented vehicle dynamics for control and estimation of distributed parameter systems

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Outline

mobile and scheduled actuators and/or sensors:

- improved estimation and control of PDEs
- better address effects of spatiotemporally varying disturbances
- requires the solution to large scale Riccati equations
- added complexity when vehicle dynamics are accounted for

address the design complexity:

- consider simpler structure of controller or filter
- minimal design complexity and computational requirements
- link vehicle motion to performance of controller/filter

main contribution:

- actuating/sensing devices affixed on vehicles that are capable of moving throughout the interior of spatial domain
- can dispense control signal/obtain process information at any point within spatial domain
- vehicle motion dictated by controller/filter performance

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Outline

We consider the state regulation of the diffusion PDE

$$\begin{aligned} \frac{\partial x(t,\xi)}{\partial t} &= a\frac{\partial^2 x(t,\xi)}{\partial \xi^2} + b(\xi)u(t), \\ x(t,0) = x(t,\ell) &= 0, \quad x(0,\xi) = x_0(\xi), \end{aligned} \tag{1}$$

- for $\xi \in \Omega = [0,\ell]$ and $t \in \mathbb{R}^+$
- $b(\xi)$: spatial distribution of actuating device
- $u(t)$: associated control signal
- $b(\xi) = \delta(\xi - \theta)$: spatial delta function with centroid at $\theta \in \Omega$
- moving actuator: centroid is time varying $b(\xi;\theta(t)) = \delta(\xi - \theta(t))$

control tasks

1. how to choose the controller architecture
2. how to choose the actuator guidance

- a way to address the first task is to use a static output feedback in which case a sensing device is collocated to the actuating device
- controller structure takes the form

$$u(t) = -\kappa x(t, \theta(t)) = -\kappa \int_0^\ell \delta(\xi - \theta(t)) x(t, \xi) \, d\xi \tag{2}$$

- $\kappa > 0$ is the feedback (scalar) gain
- $x(t, \theta)$ is interpreted as the value of the state at the spatial location $\theta(t)$

closed loop system is re-written as

$$\Sigma_1 \begin{cases} \dfrac{\partial x(t,\xi)}{\partial t} = a\dfrac{\partial^2 x(t,\xi)}{\partial \xi^2} + \delta(\xi - \theta(t))u(t), \\ x(t,0) = x(t,\ell) = 0, \\ x(0,\xi) = x_0(\xi), \\ u(t) = -\kappa \displaystyle\int_0^\ell \delta(\xi - \theta(t))x(t,\xi)\,d\xi. \end{cases}$$

addressing the second task

1. derivation of the variation of $\theta(t)$
2. include dynamics of the mobile actuator

2nd order dynamics of the mobile actuator are assumed

$$m\ddot{\theta}(t) + d\dot{\theta}(t) + k\theta(t) = f(t), \qquad \theta(0) = \theta_0, \dot{\theta}(0) = 0, \tag{3}$$

- $\theta(t)$ denotes the position of the mobile actuator
- $f(t)$ denotes the control force

redefine control objective

incorporating the vehicle dynamics, it translates to choosing the control force input $f(t)$ so that the following system is stable

$$\Sigma_2 \begin{cases} \dfrac{\partial x(t,\xi)}{\partial t} = a\dfrac{\partial^2 x(t,\xi)}{\partial \xi^2} + \delta(\xi - \theta(t))u(t), \\ x(t,0) = x(t,\ell) = 0,\ x(0,\xi) = x_0(\xi), \\ u(t) = -\kappa \displaystyle\int_0^\ell \delta(\xi - \theta(t))x(t,\xi)\,d\xi, \\ m\ddot{\theta}(t) + d\dot{\theta}(t) + k\theta(t) = f(t), \quad \theta(0) = \theta_0,\ \dot{\theta}(0) = 0. \end{cases}$$

Outline

We consider the state estimation of the diffusion PDE

$$
\begin{aligned}
&\frac{\partial x(t,\xi)}{\partial t} = a\frac{\partial^2 x(t,\xi)}{\partial \xi^2},\\
&x(t,0) = x(t,\ell) = 0, \quad x(0,\xi) = x_0(\xi),\\
&y(t) = \int_0^L c(\xi)x(t,\xi)\,\mathrm{d}\xi
\end{aligned}
\tag{4}
$$

- for $\xi \in \Omega = [0,\ell]$ and $t \in \mathbb{R}^+$
- $c(\xi)$: spatial distribution of sensing device
- $c(\xi) = \delta(\xi - \theta)$: spatial delta function with centroid at $\theta \in \Omega$
- moving sensor: centroid is time varying $c(\xi; \theta(t)) = \delta(\xi - \theta(t))$

filter tasks

1. how to choose the filter architecture
2. how to choose the sensor guidance

- a way to address the first task is to use a collocated output injection
- filter structure takes the form

$$\frac{\partial \widehat{x}(t,\xi)}{\partial t} = a\frac{\partial^2 \widehat{x}(t,\xi)}{\partial \xi^2} - \kappa\delta(\xi-\theta(t))\left(\widehat{y}(t)-y(t)\right),$$
$$\widehat{x}(t,0)=\widehat{x}(t,\ell)=0, \quad \widehat{x}(0,\xi)=\widehat{x}_0(\xi)\neq x_0(\xi), \tag{5}$$
$$\widehat{y}(t)=\int_0^L \delta(\xi-\theta(t))\widehat{x}(t,\xi)\,\mathrm{d}\xi=\widehat{x}(t,\theta(t))$$

- $\kappa > 0$ is the filter (scalar) gain
- $x(t,\theta)$ is interpreted as the value of the state at the spatial location $\theta(t)$

consider the state error $e(t,\xi) = x(t,\xi) - \widehat{x}(t,\xi)$ –error system is

$$\Sigma_3 \begin{cases} \dfrac{\partial e(t,\xi)}{\partial t} = a\dfrac{\partial^2 e(t,\xi)}{\partial \xi^2} - \kappa\delta(\xi - \theta(t))\varepsilon(t), \\ e(t,0) = e(t,L) = 0, \\ e(0,\xi) = e_0(\xi) \neq 0, \\ \varepsilon(t) = \displaystyle\int_0^L \delta(\xi - \theta(t)) e(t,\xi)\, \mathrm{d}\xi = \widehat{x}(t,\theta(t)) \end{cases}$$

addressing the second task

1. derivation of the variation of $\theta(t)$
2. include dynamics of the mobile sensor

2nd order dynamics of the mobile sensor are assumed

$$m\ddot{\theta}(t) + d\dot{\theta}(t) + k\theta(t) = f(t), \quad \theta(0) = \theta_0, \ \dot{\theta}(0) = 0, \tag{6}$$

- $\theta(t)$ denotes the position of the mobile actuator
- $f(t)$ denotes the control force

redefine filter objective

incorporating the vehicle dynamics, it translates to choosing the control force input $f(t)$ so that the following error system is stable

$$\Sigma_4 \begin{cases} \dfrac{\partial e(t,\xi)}{\partial t} = a\dfrac{\partial^2 e(t,\xi)}{\partial \xi^2} + \delta(\xi - \theta(t))\varepsilon(t), \\ e(t,0) = e(t,\ell) = 0,\ e(0,\xi) = e_0(\xi), \\ \varepsilon(t) = -\kappa \displaystyle\int_0^\ell \delta(\xi - \theta(t))e(t,\xi)\, d\xi, \\ m\ddot{\theta}(t) + d\dot{\theta}(t) + k\theta(t) = f(t),\ \theta(0) = \theta_0,\ \dot{\theta}(0) = 0. \end{cases}$$

- bring the system in abstract form

$$\begin{aligned} \dot{x}(t) &= \mathcal{A}x(t) + \mathcal{B}(\theta(t))u(t) \\ &= \Big(\mathcal{A} - \mathcal{B}(\theta(t))\kappa\mathcal{B}^*(\theta(t))\Big) = \mathcal{A}_{cl}(\theta(t))x(t) \end{aligned}$$

- *Guidance based on Lyapunov function $V(t)$:*

$$V(t) = -\frac{1}{2}\langle x(t), \mathcal{A}_{cl}(\theta(t))x(t)\rangle + \frac{1}{2}m\dot{\theta}^2(t) + \frac{1}{2}k\theta^2(t).$$

- resulting process performance-based vehicle control

$$f(t) = -x(t,\theta)x_\xi(t,\theta) - \gamma\dot{\theta}(t), \quad \gamma \geq 0$$

closed loop equations for above choice of the Lyapunov function

$$\Sigma_{cl}\begin{cases}\dfrac{\partial x(t,\xi)}{\partial t}=a\dfrac{\partial^2 x(t,\xi)}{\partial \xi^2}+\delta(\xi-\theta(t))u(t),\\ x(t,0)=x(t,\ell)=0,\\ x(0,\xi)=x_0(\xi),\\ u(t)=-\kappa\displaystyle\int_0^\ell \delta(\xi-\theta(t))x(t,\xi)\,d\xi,\\ m\ddot{\theta}(t)+d\dot{\theta}(t)+k\theta(t)=f(t),\quad \theta(0)=\theta_0,\ \dot{\theta}(0)=0\\ f(t)=-x(t,\theta)x_\xi(t,\theta)-\gamma\dot{\theta}(t),\ \gamma\geq 0.\end{cases}$$

Remark

The vehicle control force $f(t)$ requires the signals $x(t,\theta)$, $x_\xi(t,\theta)$ and $\dot{\theta}(t)$. The signal $x(t,\theta)$ is the output $y(t;\theta(t))$ and $x_\xi(t,\theta)$ is the spatial derivative of the output $y(t;\theta(t))$. For compact notation, we adopt $y_\xi(t;\theta(t)) = x_\xi(t,\theta(t))$ with the understanding that

$$y_\xi(t;\theta(t)) = \frac{\partial x(t,\xi)}{\partial \xi}\Big|_{\xi=\theta(t)}$$

Finally, it is assumed that the vehicle knows its own state $(\theta,\dot{\theta})$ and therefore the velocity $\dot{\theta}(t)$ is assumed to be available. Then using the above notation, the expression for the control force can be compactly written as

$$f(t) = -y(t;\theta(t))y_\xi(t;\theta(t)) - \gamma\dot{\theta}(t),$$

and which requires 3 scalar signals $y(t;\theta(t)), y_\xi(t;\theta(t)), \dot{\theta}(t)$ to be realized

Lemma

Consider the system (4) *with the control law* (2). *Assume that the vehicle dynamics that describe the motion of the actuator centroid* $\theta(t)$ *are described by* (6) *and that the vehicle knows its own state* $(\theta, \dot{\theta})$. *Then the proposed Lyapunov-based vehicle+actuator guidance law* $f_l(t)$ *renders the system* Σ_2 *stable.*

Remark

Similar results can be obtained for the moving sensor in the filter case.

Remark (case of multiple moving actuators (N agents))

$$
\Sigma_{cl}\left\{
\begin{array}{l}
\dfrac{\partial x(t,\xi)}{\partial t} = a\dfrac{\partial^2 x(t,\xi)}{\partial \xi^2} + \displaystyle\sum_{i=1}^{N} \delta(\xi - \theta_i(t))u_i(t), \\
x(t,0) = x(t,\ell) = 0, \\
x(0,\xi) = x_0(\xi), \\
u_i(t) = -\displaystyle\sum_{i=1}^{N} \kappa_{ij} \int_0^\ell \delta(\xi - \theta_i(t))x(t,\xi)\, d\xi, \;\; i = 1,\dots,N \\
m_i\ddot{\theta}_i(t) + d_i\dot{\theta}_i(t) + k_i\theta_i(t) = f_i(t), \quad \theta_i(0) = \theta_{0i}, \; \dot{\theta}_i(0) = 0 \\
\left.\begin{array}{rcl}
f_i(t) & = & -x_\xi(t,\theta_i)\displaystyle\sum_{i=1}^{N} \kappa_{ij} x(t,\theta_j(t)) - \displaystyle\sum_{i=1}^{N} \gamma_{ij}\dot{\theta}_j(t) \\
 & = & x_\xi(t,\theta_i)u_i(t) - \displaystyle\sum_{i=1}^{N} \gamma_{ij}\dot{\theta}_j(t)
\end{array}\right. , \;\; i = 1,\dots,N
\end{array}
\right.
$$

Remark (case of adaptive feedback gain-N agents)

$$
\Sigma_{cl}
\begin{cases}
\dfrac{\partial x(t,\xi)}{\partial t} = a\dfrac{\partial^2 x(t,\xi)}{\partial \xi^2} + \displaystyle\sum_{i=1}^{N} \delta(\xi - \theta_i(t))u_i(t), \\
x(t,0) = x(t,\ell) = 0, \\
x(0,\xi) = x_0(\xi), \\
u_i(t) = -\displaystyle\sum_{i=1}^{N} \kappa_{ij}(t) \int_0^{\ell} \delta(\xi - \theta_i(t))x(t,\xi)\, d\xi, \;\; i = 1,\dots,N \\
m_i\ddot{\theta}_i(t) + d_i\dot{\theta}_i(t) + k_i\theta_i(t) = f_i(t), \quad \theta_i(0) = \theta_{0i},\; \dot{\theta}_i(0) = 0 \\
f_i(t) = x_{\xi}(t,\theta_i)u_i(t) - \displaystyle\sum_{i=1}^{N} \gamma_{ij}\dot{\theta}_j(t), \;\; i = 1,\dots,N \\
\dot{\kappa}_{ij}(t) = -y_i(t)y_j(t), \;\; i,j = 1,\dots,N
\end{cases}
$$

Remark (2D case)

$$\begin{cases} \dfrac{\partial x(t,\xi,\psi)}{\partial t} = a\left(\dfrac{\partial^2 x(t,\xi,\psi)}{\partial \xi^2} + \dfrac{\partial^2 x(t,\xi,\psi)}{\partial \psi^2}\right) + \sum_{i=1}^{N} b_i(\xi,\psi)u_i(t), \\ x(t,\cdot,\cdot)\Big|_{\partial\Omega} = 0, \quad x(0,\xi,\psi) = x_0(\xi,\psi), \\ u_i(t) = -\sum_{j=1}^{N} \kappa_{ij}(t) \int_0^{L_\xi}\int_0^{L_\psi} b_i(\xi,\psi)x(t,\xi,\psi)\,\mathrm{d}\psi\mathrm{d}\xi = -\sum_{j=1}^{N} \kappa_{ij}(t)y_j(t), \\ b_i(\xi,\psi) = \delta(\xi - \xi_i(t))\delta(\psi - \psi_i(t)), \\ \dot{q}_i(t) = S(q_i)v_i(t), \;\; q_i(t) = (\xi_i(t), \psi_i(t), \theta_i(t)) \\ M_i\dot{v}_i(t) = B_i\tau_i(t), \qquad i = 1,\dots,N. \end{cases}$$

Remark (2D case)

- *generalized coordinates vector* $q_i(t)$ *consisting of horizontal distance* $\xi_i(t)$, *vertical distance* $\psi_i(t)$ *and orientation* $\theta_i(t)$
- $v_i(t) = (\upsilon_i(t), \omega_i(t))$: $\upsilon_i(t)$ *and* $\omega_i(t)$ *the linear and angular velocities*
- $S(q_i)$: *mobile base coordinates* $v_i(t)$ *to Cartesian coordinates* $q_i(t)$

$$S(q_i) = \begin{bmatrix} \cos(\theta_i) & -d\sin(\theta_i) \\ \sin(\theta_i) & d\cos(\theta_i) \\ 0 & 1 \end{bmatrix}$$

- *(vehicle guidance):* $\lambda > 0$ *a guidance gain,* $K = K^T = \{\kappa_{ij}\} > 0$

$$\tau_i(t) = \lambda B_i^{-1} S^T(q_i)\Big(\Psi_i(t) u_i(t)\Big) - B_i^{-1}\sum_{j=1}^{N} \gamma_{ij} v_j(t), \quad \Psi_i(t) \triangleq \begin{bmatrix} x_\xi(t,\xi_i,\psi_i) \\ x_\psi(t,\xi_i,\psi_i) \\ 0 \end{bmatrix}$$

Outline

1. Introduction-motivation
2. Problem statement
   - Mobile actuators
   - Mobile sensors
3. Guidance of moving collocated actuators/sensors
4. Numerical results
   - 1D diffusion equation
   - 2D diffusion equation
5. Conclusions

- PDE with 80 linear elements in $\Omega = [0,1]$ and $x(0,\xi) = \sin(\pi\xi/L)e^{-7\xi^2}$
- coefficient of spatial operator: $a = 0.005$
- moving source was taken as a spatial delta function with constant intensity and a moving centroid $\xi_s(t)$

$$d(t,\xi) = 2\times 10^{-3}\delta(\xi - \xi_s(t)),\ \xi_s(t) = 0.3\ell(\cos(\frac{2\pi t}{t_f}) + 2).$$

- vehicle parameters $m = 1, k = 1, d = \sqrt{2}$ with $\theta(0) = 0.25\ell, \dot{\theta}(0) = 0$
- static feedback gain was chosen as $\kappa = 100$
- implemented $f(t) = \alpha y(t;\theta(t))y_\xi(t;\theta(t)) - \gamma\dot{\theta}(t)$, $\alpha = 1$, $\gamma = 0.05 - d$
- closed loop system was simulated in the time interval $[t_0, t_f] = [0, 20]s$

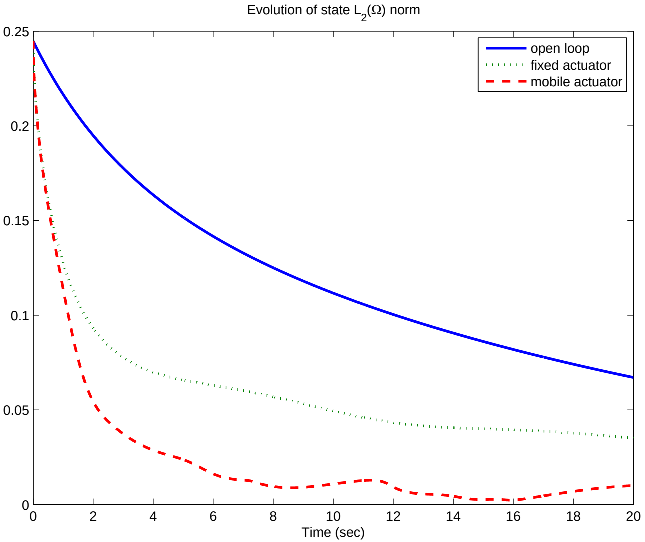

Figure: Evolution of $L_2(\Omega)$ norms.

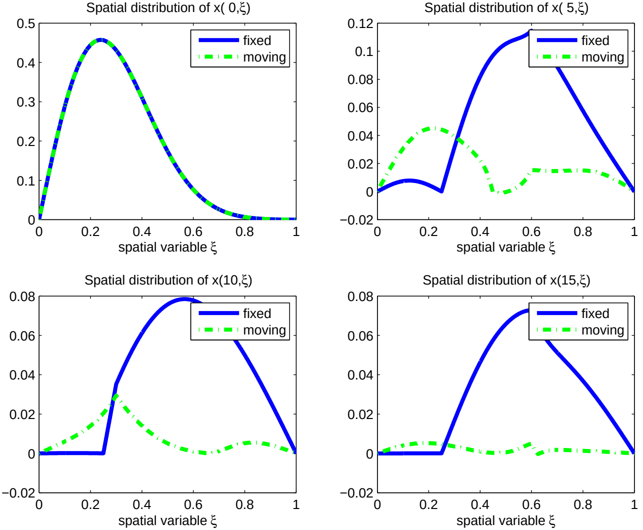

Figure: Closed loop state vs spatial variable at different time instances.

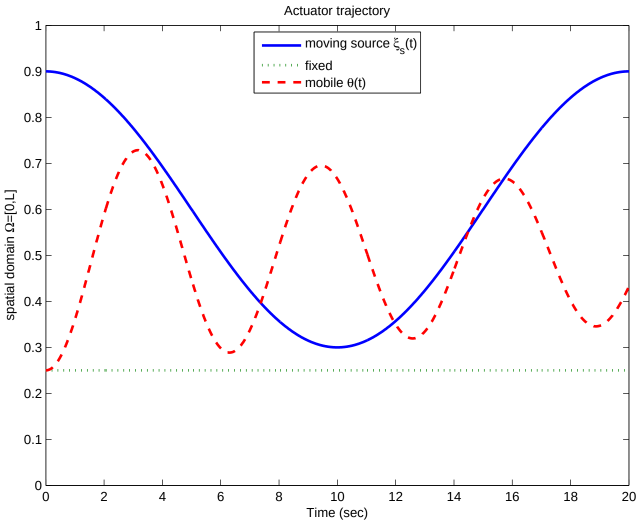

Figure: Evolution of actuator and disturbance trajectories.

Outline

- FEM scheme, 30 elements/direction, $[0, L_\xi] \times [0, L_\psi] = [0, 100] \times [0, 60]$
- I.C. $x(0, \xi, \psi) = 25 \times 10^4 \left(\frac{\xi}{L_\xi}\right)^3 \left(1 - \frac{\xi}{L_\xi}\right)^3 \left(\frac{\psi}{L_\psi}\right)^3 \left(1 - \frac{\psi}{L_\psi}\right)^3$
- coefficient of spatial operator: $a = 10$
- terrain vehicle with $m = 9$, $I = 0.624$, $R = 0.1$, $r = 0.05$ and $d = 0.01$
- $\xi_1(0) = 0.312 L_\xi, \psi_1(0) = 0.123 L_\psi, \theta_1(0) = \pi, v_1(0) = (4, 0)$
- adaptive feedback gain, I.C. $\kappa_1(0) = 10^9$, guidance gain $\lambda = 0.01$
- a "moving" disturbance was included as an added input, given by

$$d(t, \xi, \psi) = 10^{-2} \delta(\xi - \xi_s(t)) \delta(\psi - \psi_s(t)),$$

- spatial distribution of the moving source: 2D delta function

$$\xi_s(t) = L_\xi \left[0.5 - 0.45 \sin\left(\frac{5\pi t}{t_f}\right)\right], \quad \psi_s(t) = L_\psi \left[0.5 - 0.45 \cos\left(\frac{3\pi t}{t_f}\right)\right],$$

- C.L. system simulated on time interval $[t_0, t_f] = [0, 30]$

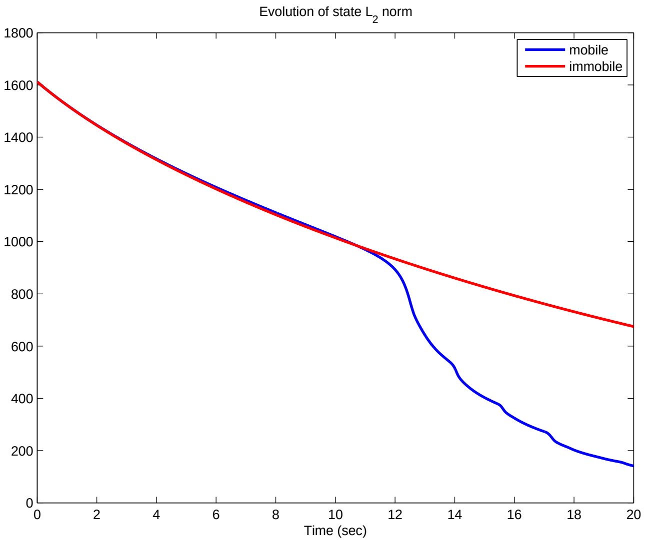

Figure: Comparison of mobile vs fixed actuator/sensor pair: Evolution of L_2 norm.

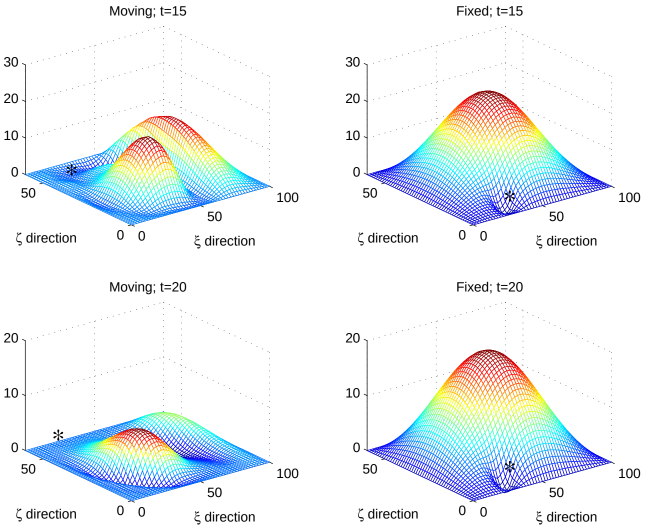

Figure: Comparison of mobile vs fixed actuator/sensor pair: Spatial state distribution.

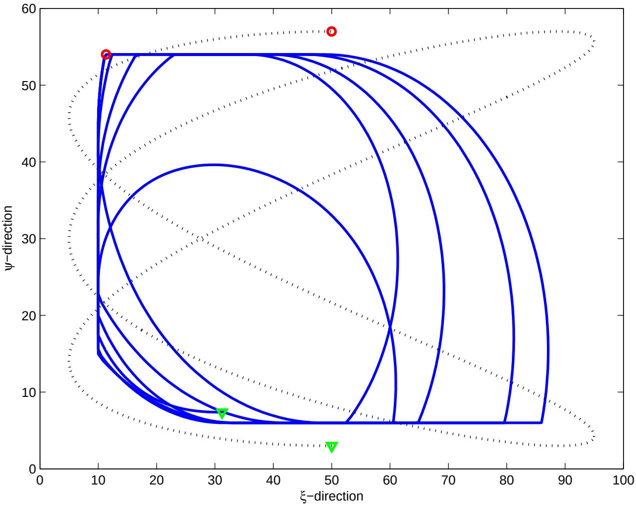

Figure: Mobile actuator/sensor trajectory (blue solid); moving source trajectory (black dotted). Start position ▽; end position ○.

- proposed a stability-based scheme for the guidance of a mobile actuator used for performance enhancement of a class of PDEs
- Lyapunov-based scheme included the mobile agent dynamics
- analytical expression for the motion of the centroid of the moving actuators/sensors
- use of multiple mobile actuators/multiple sensors
- motion coordination of multiple vehicles with collision avoidance modifications and localization algorithms for estimating the state of each vehicle