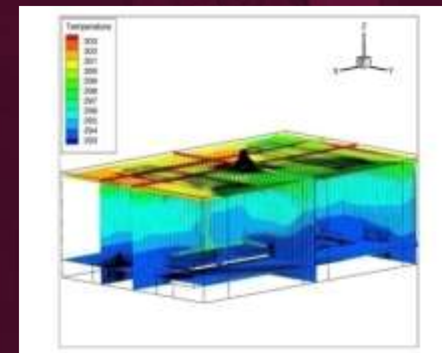
Feedback Control of Boussinesq Equations with Applications to Energy Efficient Buildings

Optimal Sensor Location



Room with Disturbance

VT – I. Akhtar, J. Borggaard, J. Burns, E. Cliff, W. Hu, L. Zietsman

UTRC – S. Ahuja, S. Narayanan, A. Surana

VirginiaTech
Invent the Future

ICAM

Interdisciplinary Center for Applied Mathematics

CDPS 2011
Wuppertal, Germany
18 – 22 July 2011

Energy Efficiency & Buildings

WHY BUILDINGS

Energy Efficiency & Buildings

WHY BUILDINGS

AND NOT THIS

Building Energy Demand

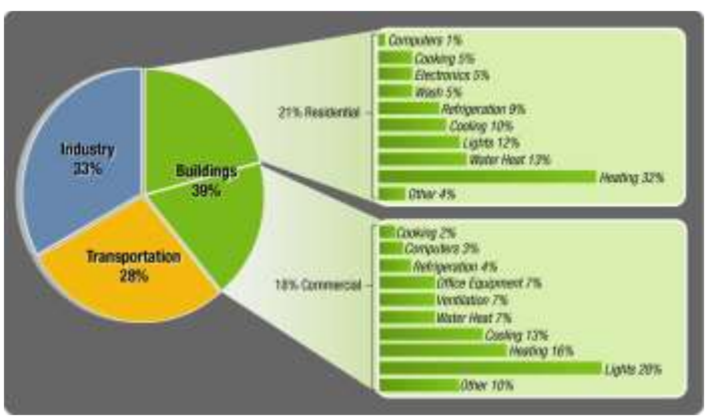
Buildings consume
- **39% of total U.S. energy**
- **71% of U.S. electricity**
- **54% of U.S. natural gas**

Building produce 48% of U.S. Carbon emissions

Commercial building annual energy bill: $120 billion

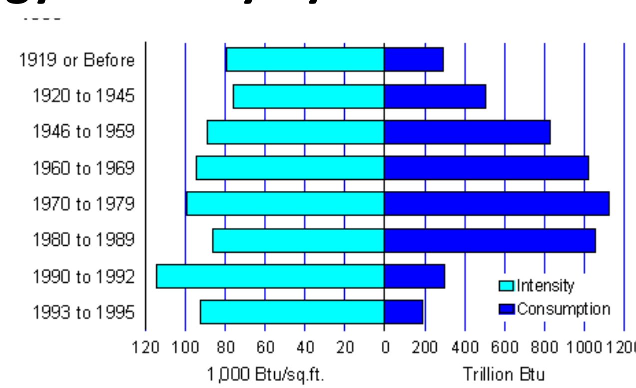
The *only* energy end-use sector showing growth in energy intensity
- **17% growth 1985 - 2000**
- **1.7% growth projected through 2025**

Energy Breakdown by Sector



Sources: Ryan and Nicholls 2004, USGBC, USDOE 2004

Energy Intensity by Year Constructed



Energy Information Administration
1995 Commercial Buildings Energy Consumption Survey

Impact of Energy Efficient Buildings

HUGE

- A 50 percent reduction in buildings' energy usage would be equivalent to taking every passenger vehicle and small truck in the United States off the road.
- A 70 percent reduction in buildings' energy usage is equivalent to eliminating the entire energy consumption of the U.S. transportation sector.

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DESIGN, CONTROL AND OPTIMIZATION OF WHOLE BUILDING SYSTEMS IS THE ONLY WAY TO GET THERE

? WHY ?

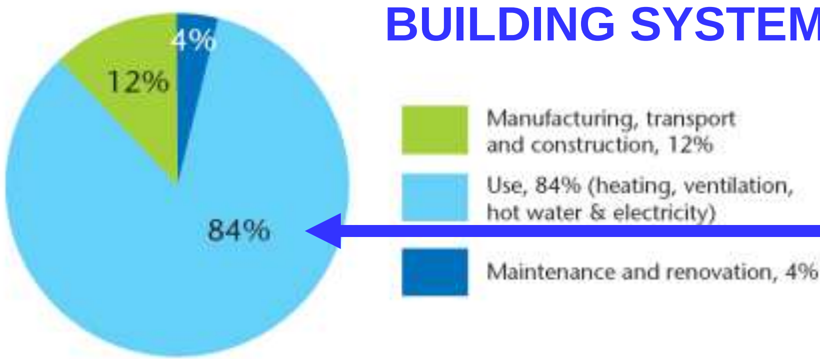


Figure 3.7: Life cycle energy use

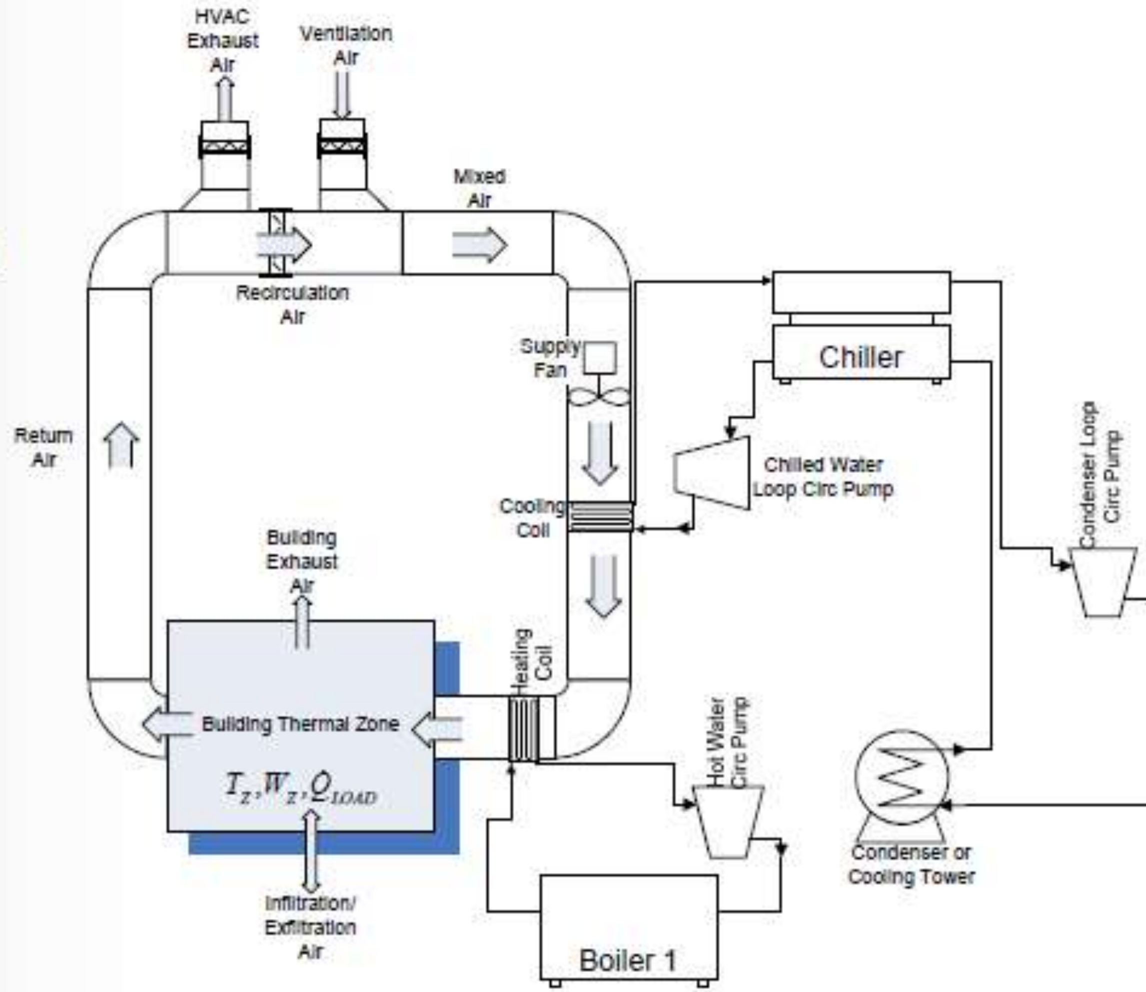
84% of energy consumed in buildings is during the use of the building

REQUIRES COMBINING - MODELING, COMPLEX MULTI-SCALE DYNAMICS, CONTROL, OPTIMIZATION, SENSITIVITY ANALYSIS, HIGH PERFORMANCE COMPUTING … ALL THE THINGS THAT APPLIED MATHEMATICIANS DO

Traditional HVAC System

Idealized Building HVAC System

- Assumptions:
  - One HVAC system simulation for single building thermal zone
  - Variable Airflow
  - Constant COPs
  - Simpler systems can be modeled by deleting components



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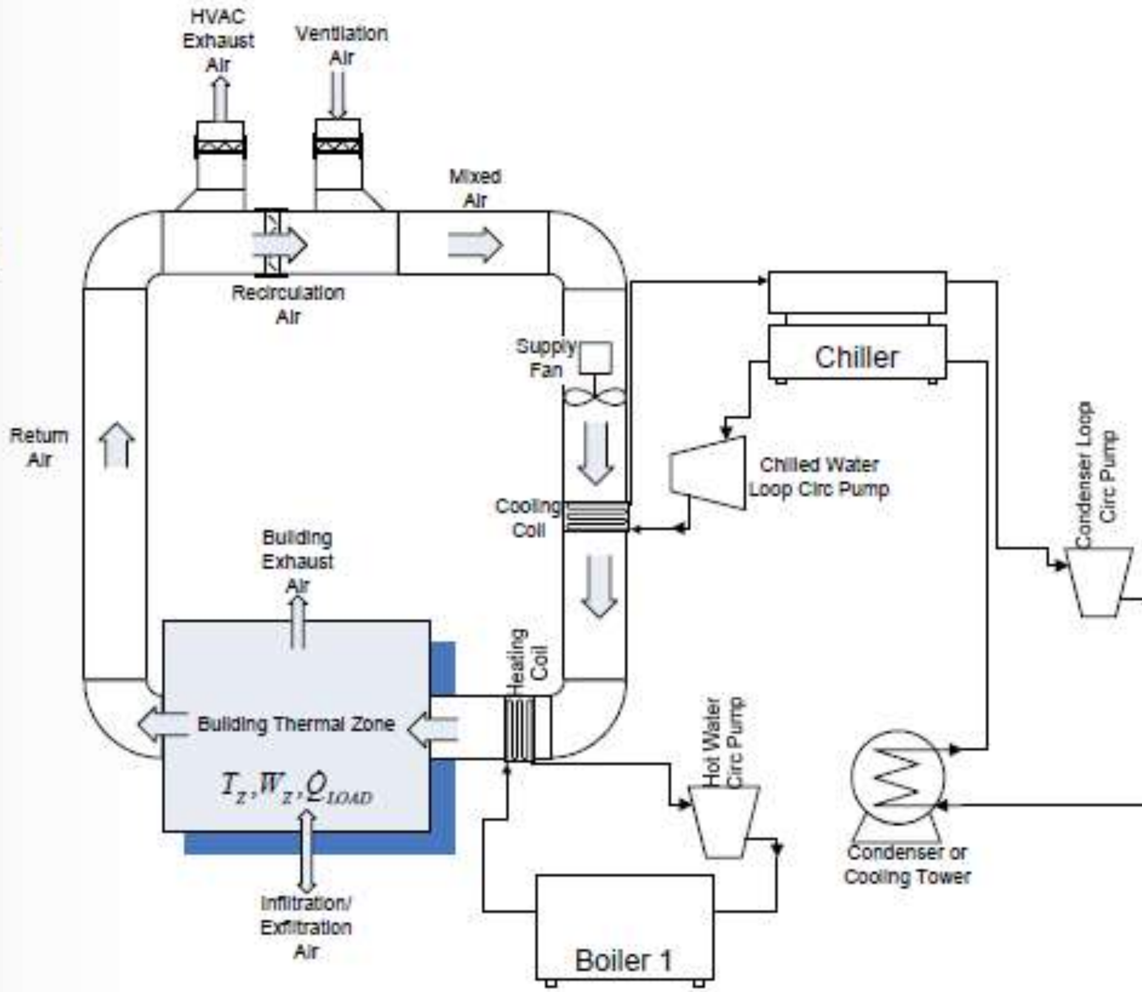

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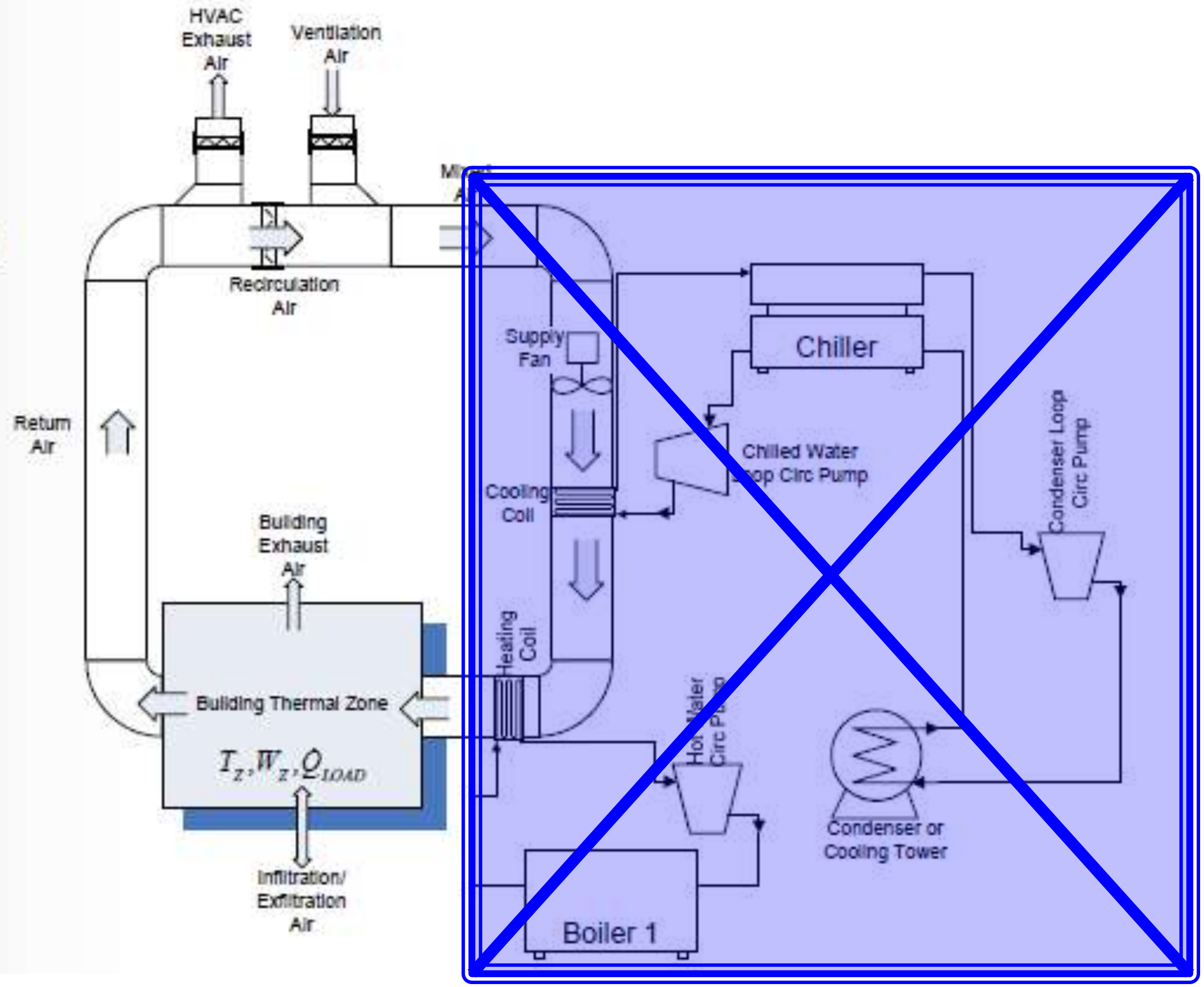

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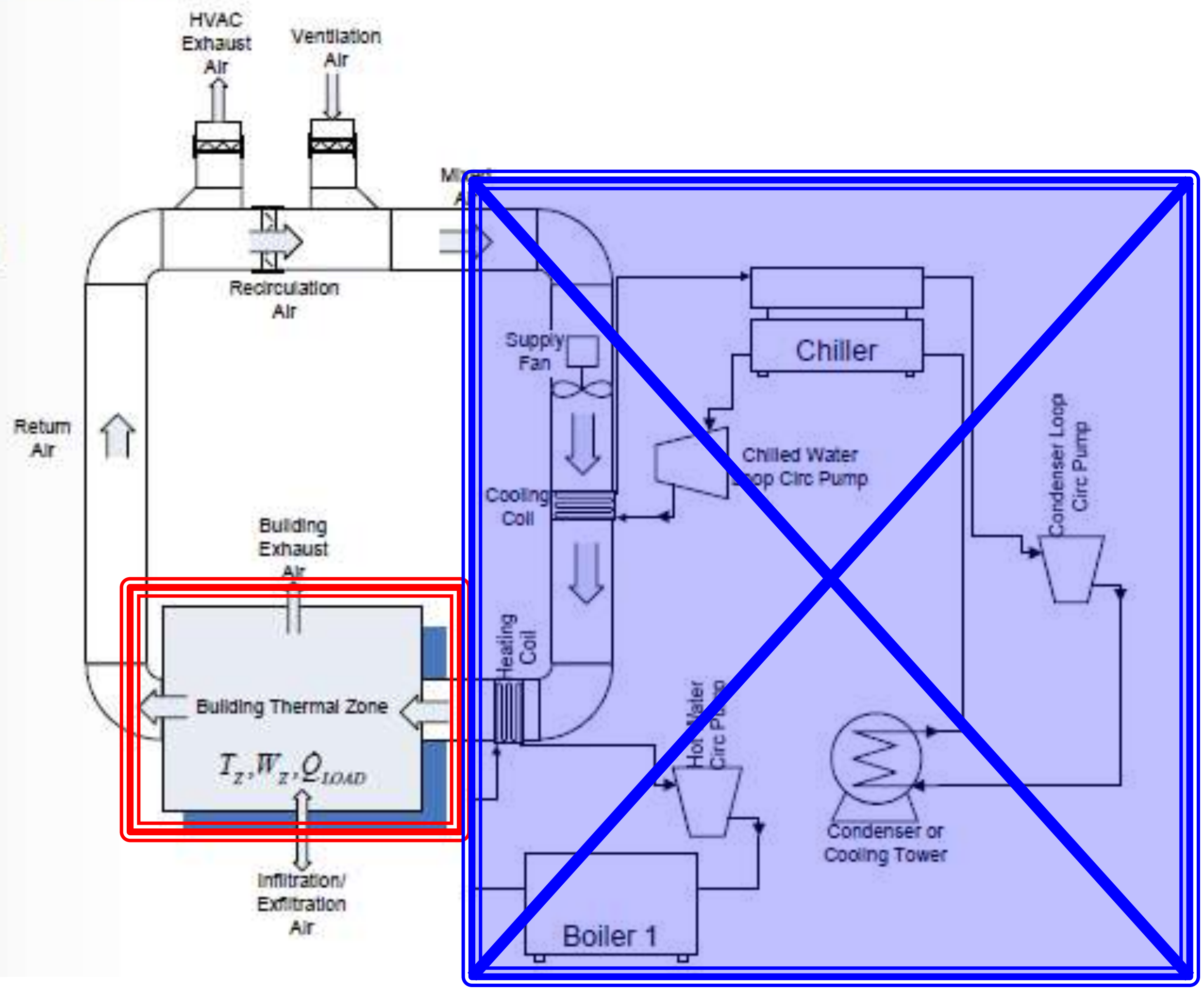
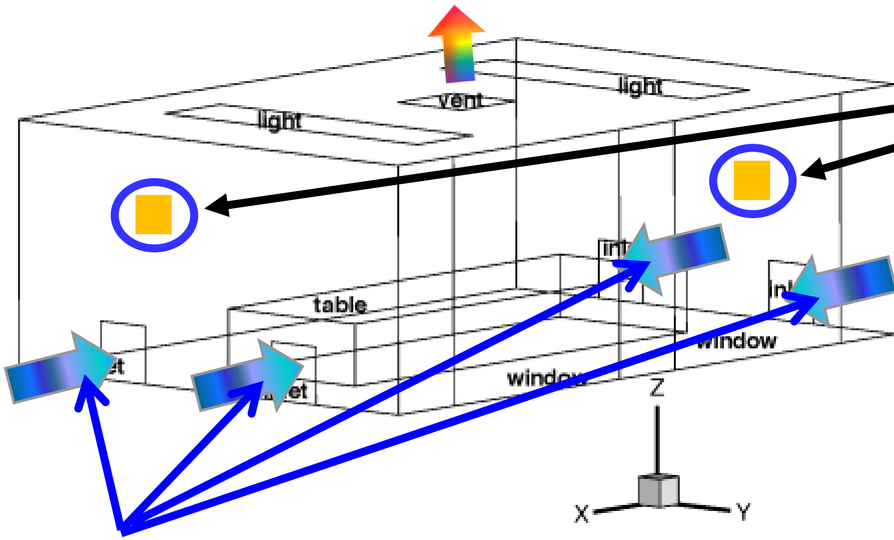


Conference Room Control Problem



sensors / region

$$\Omega(\vec{\mathbf{q}}) = \left\{ \vec{\mathbf{x}} \in \bar{\Omega} : \left\| \vec{\mathbf{q}} - \vec{\mathbf{x}} \right\| < \delta \right\}$$

controlled region is around conference table

$u_T(t)$ = inflow temperature

$$T(t,\vec{\mathbf{x}})\,|_{\Gamma_c} = b_T(\vec{\mathbf{x}})\,u_T(t)$$

$$\frac{\partial}{\partial t} T(t,\vec{\mathbf{x}}) + \mathbf{v}(t,\vec{\mathbf{x}}) \bullet \nabla T(t,\vec{\mathbf{x}}) = \kappa \nabla^2 T(t,\vec{\mathbf{x}}) + g_T(\vec{\mathbf{x}})\,w_T(t)$$

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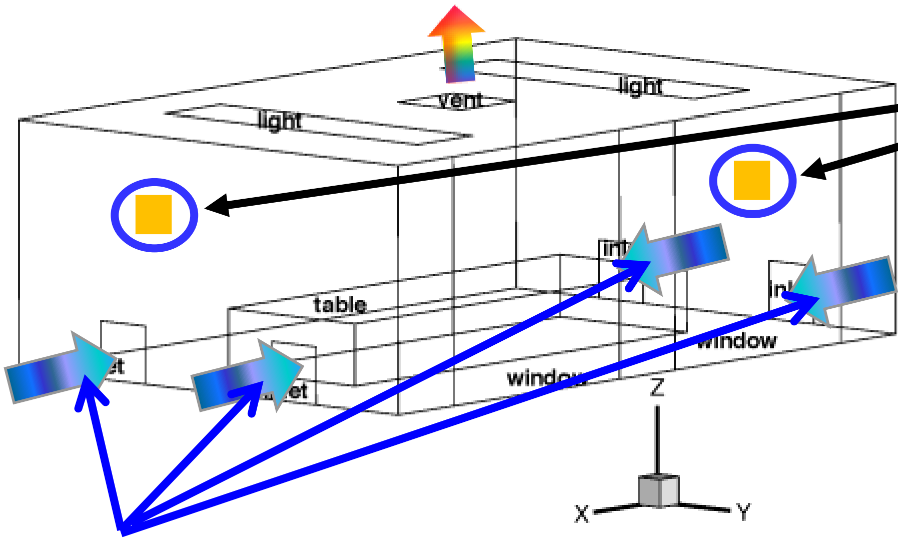

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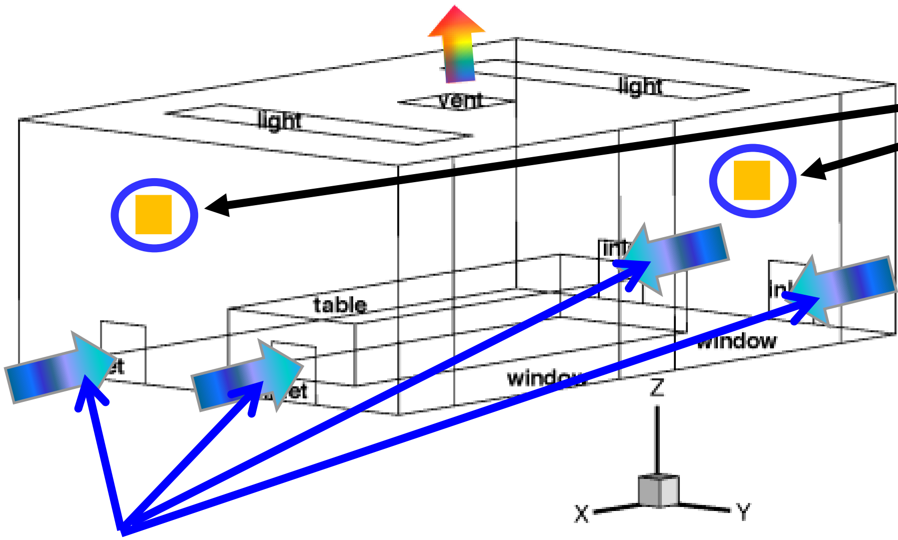
WHAT IS A “GOOD” FORMULATION OF THE CONTROL PROBLEM

ICAM

Conference Room Control Problem

Virginia Tech



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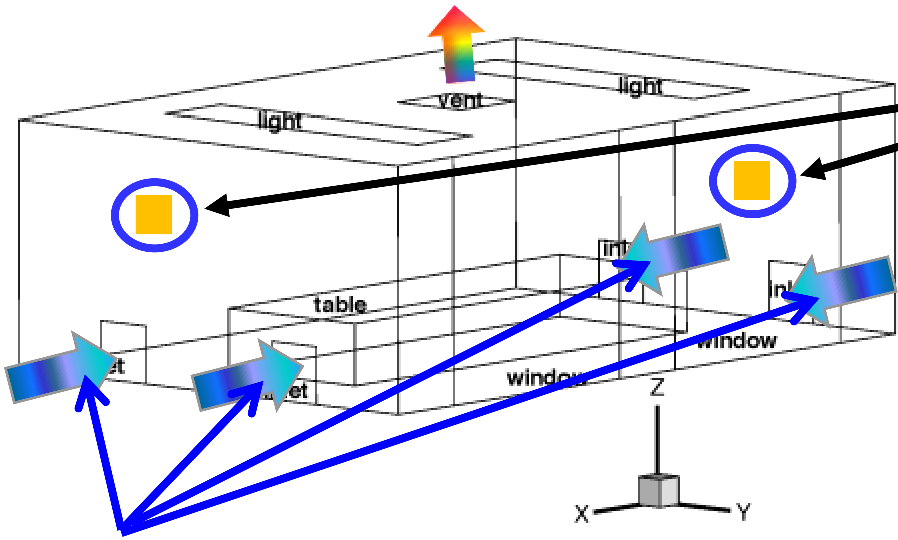
$$\xi(t) = \mathcal{D}T(t,\cdot)$$

CONTROLLED OUTPUT ??

Conference Room Control Problem



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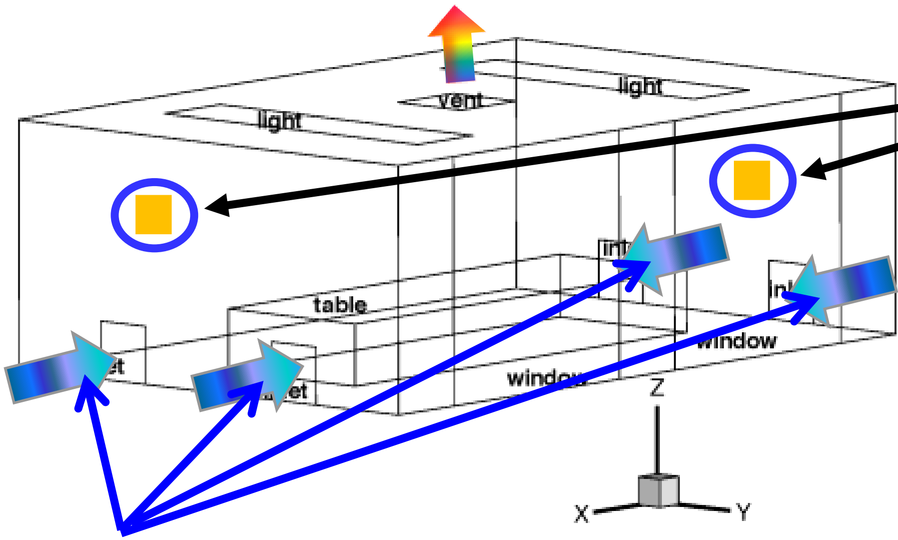
AVERAGE ROOM TEMPERATURE (WEIGHTED) ?

Conference Room Control Problem



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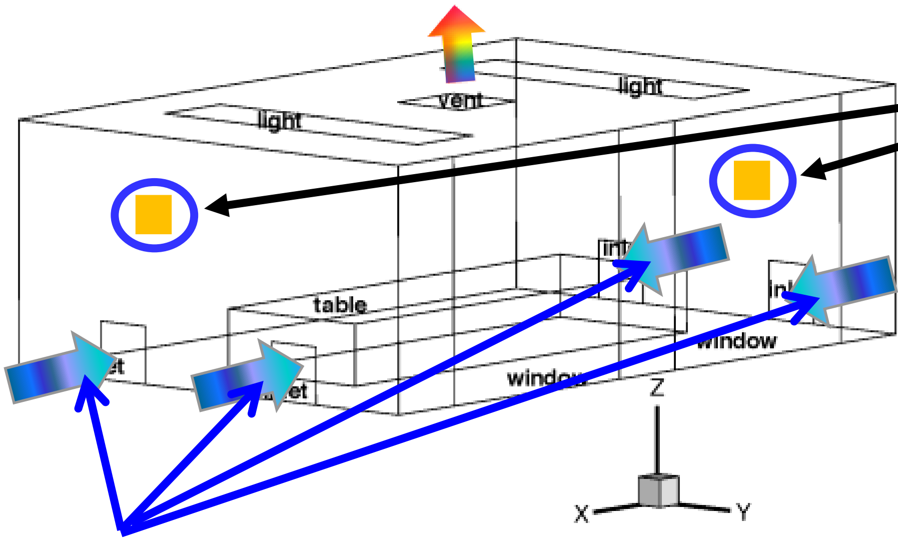
AVERAGE ROOM TEMPERATURE (WEIGHTED) ?

$$[\xi(t)](\vec{x}) = \mathcal{D}T(t,\vec{x}) = d(\vec{x})T(t,\vec{x}) \in L_2(\Omega)$$

LOCAL IN SPACE TEMPERATURE (WEIGHTED) ?

Conference Room Control Problem



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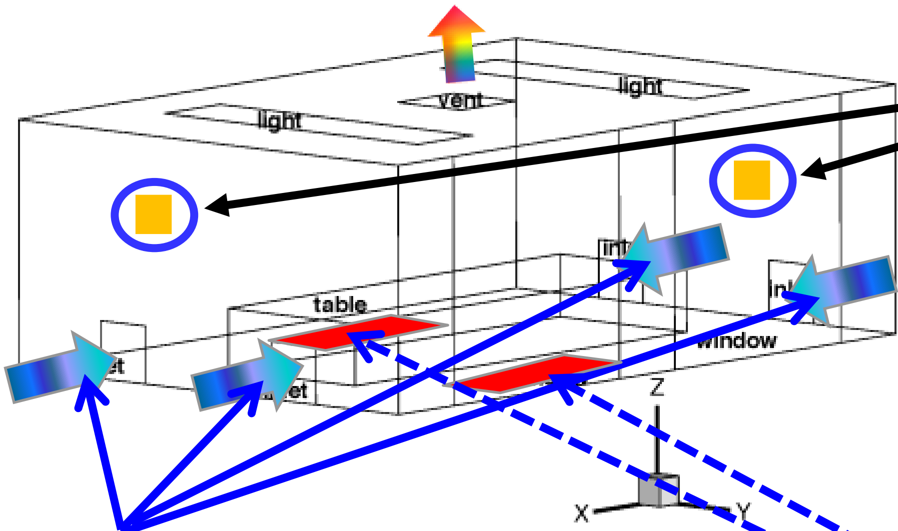
AVERAGE ROOM TEMPERATURE (WEIGHTED) ?

? WHERE SHOULD ONE PLACE SENSORS ?

Other Controls: Floor “Heating Strips”



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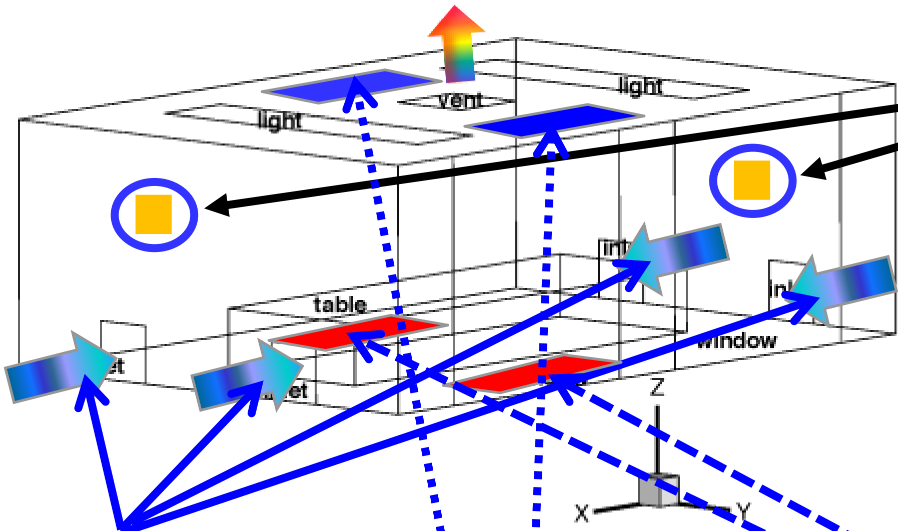
Other Controls: "Chilled Beams"



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ICAM

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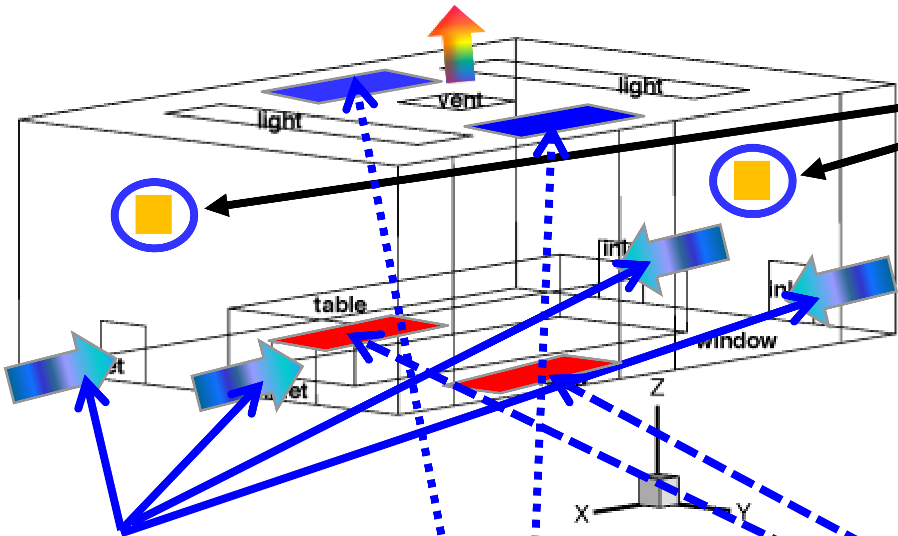
Virginia Tech



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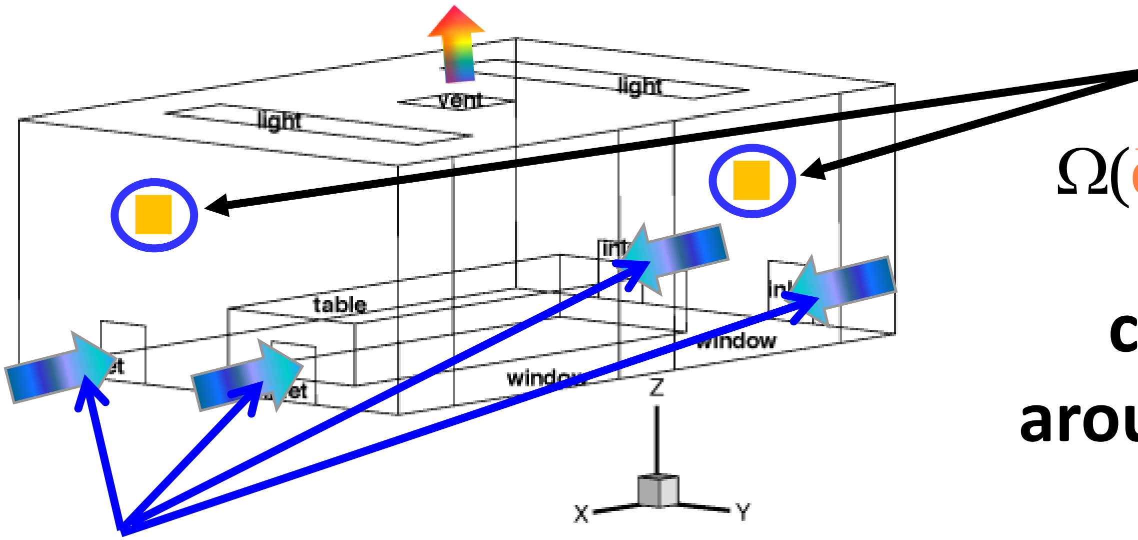
? MORE COMPLEX PHYSICS ?

Boussinesq Equations



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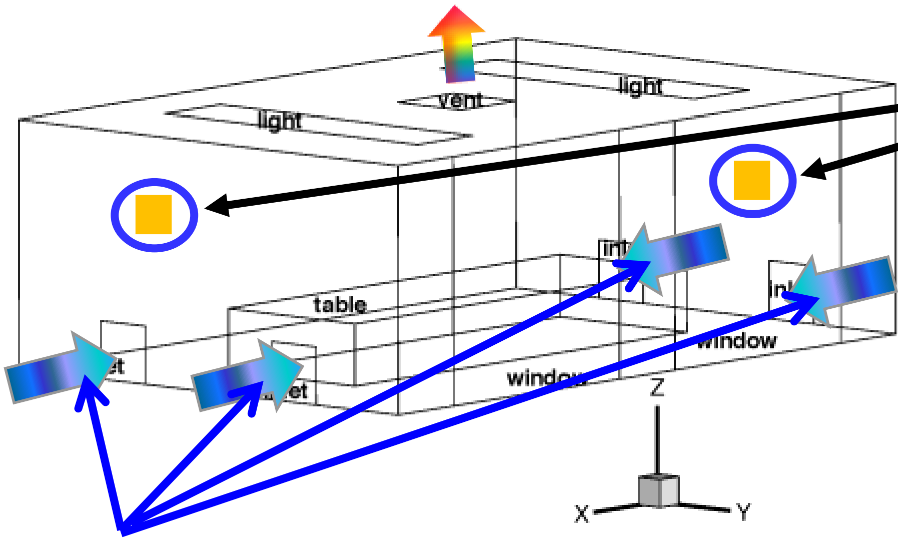
Boussinesq Equations



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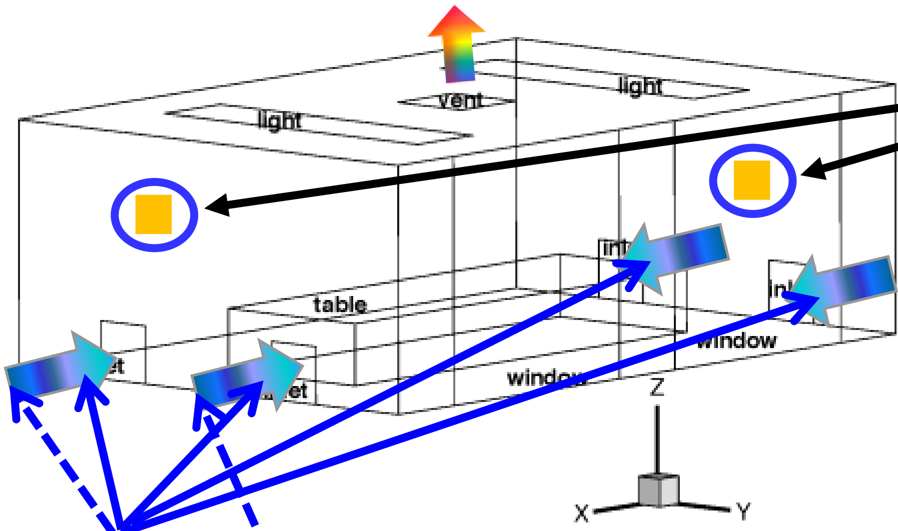
$$+ g\alpha_T e_d T(t,\vec{\mathbf{x}}) + g_{\mathbf{v}}(\vec{\mathbf{x}})w_{\mathbf{v}}(t)$$

$$div\ \mathbf{v}(t,\vec{\mathbf{x}}) = 0$$

Boussinesq Equations



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Linearized Model

$$\frac{\partial}{\partial t}T + \mathbf{v}\bullet\nabla\overline{T} + \overline{\mathbf{v}}\bullet\nabla T = \kappa\nabla^2 T + g_T w_T(t)$$

$$\frac{\partial}{\partial t}\mathbf{v} + \left(\overline{\mathbf{v}}\bullet\nabla\mathbf{v}\right) + \left(\mathbf{v}\bullet\nabla\overline{\mathbf{v}}\right) = \nu\nabla^2\mathbf{v} - \nabla p$$

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Linearized Model

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$$+ g \alpha_T e_d T + g_{\mathbf{v}} w_{\mathbf{v}}(t)$$

IN THE PROPER SPACES FOR VARIOUS CONTROL INPUTS

$$(\Sigma_T) \qquad \dot{T}(t) = \mathcal{A}_T T(t) + \mathcal{F} \mathbf{v}(t) + \mathcal{B}_T u_T(t) + \mathcal{G}_T w_T(t)$$

$$(\Sigma_{\mathbf{v}}) \qquad \dot{\mathbf{v}}(t) = \mathcal{A}_{\mathbf{V}} \mathbf{v}(t) + \alpha \Lambda T(t) + \mathcal{B}_{\mathbf{V}} u_{\mathbf{V}}(t) + \mathcal{G}_{\mathbf{V}} w_{\mathbf{V}}(t)$$

Specific Structure

$$(\Sigma) \qquad \frac{d}{dt}\begin{bmatrix} T(t) \\ \mathbf{v}(t) \end{bmatrix} = \begin{bmatrix} \mathcal{A}_T & \mathcal{F} \\ \alpha\Lambda & \mathcal{A}_{\mathbf{v}} \end{bmatrix}\begin{bmatrix} T(t) \\ \mathbf{v}(t) \end{bmatrix} + \begin{bmatrix} \mathcal{B}_T & 0 \\ 0 & \mathcal{B}_{\mathbf{v}} \end{bmatrix}\begin{bmatrix} u_T(t) \\ u_{\mathbf{v}}(t) \end{bmatrix} + \begin{bmatrix} \mathcal{G}_T & 0 \\ 0 & \mathcal{G}_{\mathbf{v}} \end{bmatrix}\begin{bmatrix} w_T(t) \\ w_{\mathbf{v}}(t) \end{bmatrix}$$

Specific Structure: $\alpha \approx 0$

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Theory Slide

$$(\Sigma_T) \qquad \dot{T}(t) = \mathcal{A}_T T(t) + \mathcal{F}\mathbf{v}(t) + \mathcal{B}_T u_T(t) + \mathcal{G}_T w_T(t)$$

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$$(\Sigma) \qquad \dot{x}(t) = \mathcal{A}x(t) + \mathcal{B}u(t) + \mathcal{G}w(t)$$

$$\mathcal{A} = \begin{bmatrix} \mathcal{A}_T & \mathcal{F} \\ \alpha\Lambda & \mathcal{A}_{\mathbf{v}} \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} \mathcal{B}_T & 0 \\ 0 & \mathcal{B}_{\mathbf{v}} \end{bmatrix}, \quad \mathcal{G} = \begin{bmatrix} \mathcal{G}_T & 0 \\ 0 & \mathcal{G}_{\mathbf{v}} \end{bmatrix}$$

Theory Slide

$$(\Sigma_T) \qquad \dot{T}(t) = \mathcal{A}_T T(t) + \mathcal{F}\mathbf{v}(t) + \mathcal{B}_T u_T(t) + \mathcal{G}_T w_T(t)$$

$$(\Sigma_{\mathbf{v}}) \qquad \dot{\mathbf{v}}(t) = \mathcal{A}_{\mathbf{v}}\mathbf{v}(t) + \alpha\Lambda T(t) + \mathcal{B}_{\mathbf{v}} u_{\mathbf{v}}(t) + \mathcal{G}_{\mathbf{v}} w_{\mathbf{v}}(t)$$

$$(\Sigma) \qquad \dot{x}(t) = \mathcal{A}x(t) + \mathcal{B}u(t) + \mathcal{G}w(t)$$

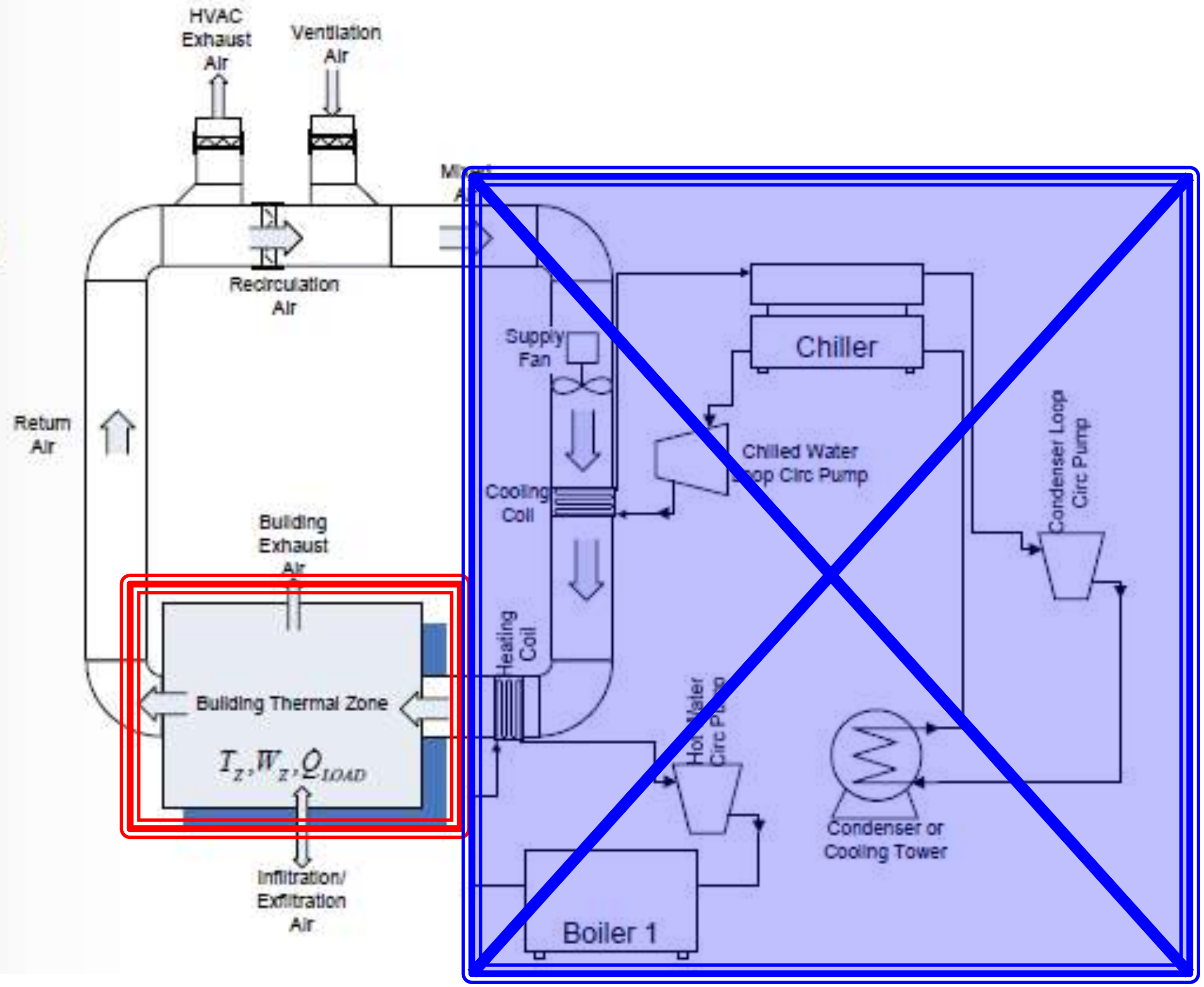
$$\mathcal{A} = \begin{bmatrix} \mathcal{A}_T & \mathcal{F} \\ \alpha\Lambda & \mathcal{A}_{\mathbf{v}} \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} \mathcal{B}_T & 0 \\ 0 & \mathcal{B}_{\mathbf{v}} \end{bmatrix}, \quad \mathcal{G} = \begin{bmatrix} \mathcal{G}_T & 0 \\ 0 & \mathcal{G}_{\mathbf{v}} \end{bmatrix}$$

Theorem (J. Burns & W. Hu) In a suitable space X, the system (Σ) is well posed. Moreover, a LQR feedback law $u(t) = -\mathcal{B}^*\Pi$ that exponentially stabilizes (Σ) also locally stabilizes the full non-linear Boussinesq equations.

Traditional HVAC System

Idealized Building HVAC System

- Assumptions:
  - One HVAC system simulation for single building thermal zone
  - Variable Airflow
  - Constant COPs
  - Simpler systems can be modeled by deleting components



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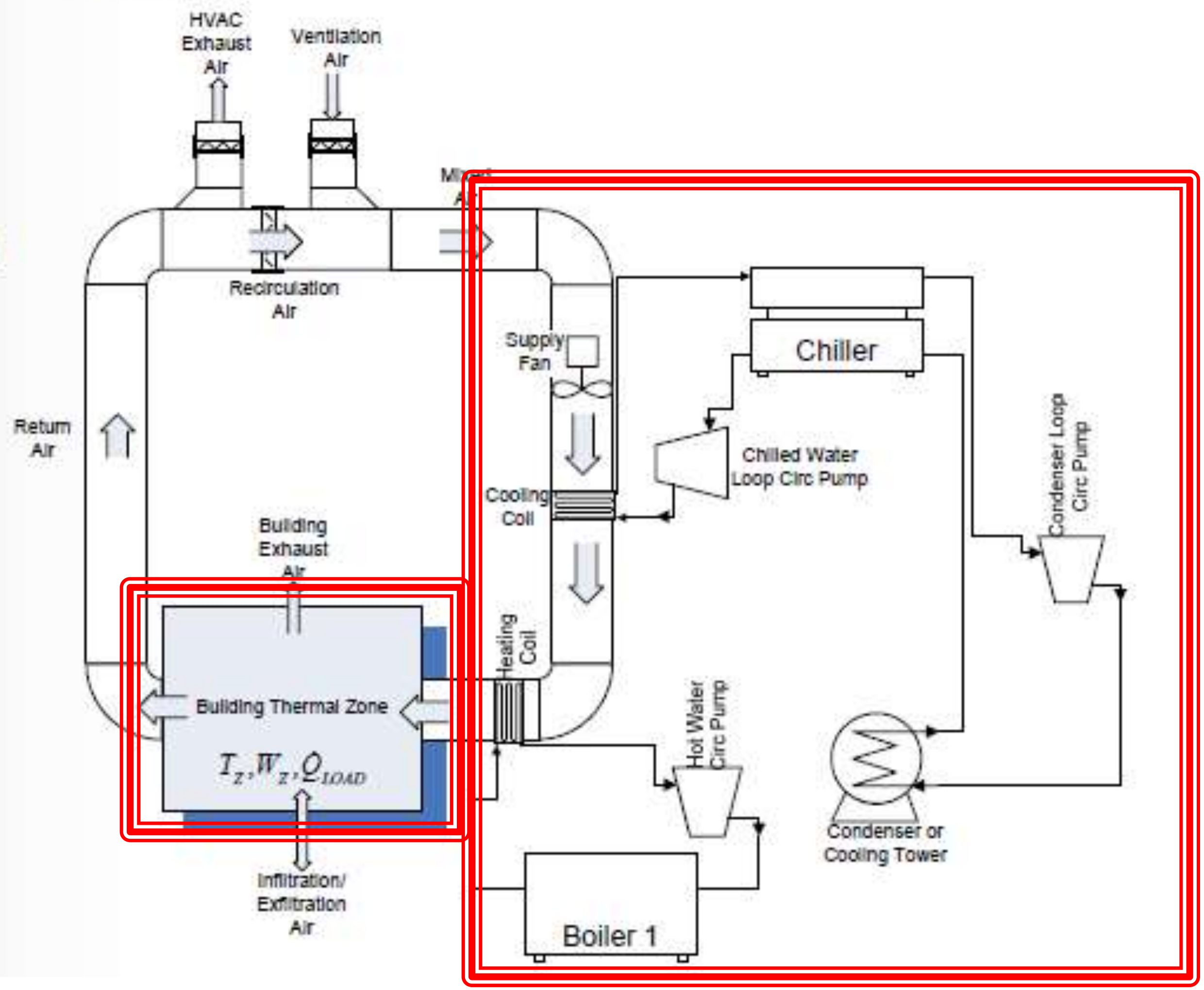

ADD "ACTUATOR" MODEL / DYNAMICS

Actuator Dynamics

$$(\Sigma) \qquad \frac{d}{dt}\begin{bmatrix} T(t) \\ \mathbf{v}(t) \end{bmatrix} = \begin{bmatrix} \mathcal{A}_T & \mathcal{F} \\ 0 & \mathcal{A}_{\mathbf{v}} \end{bmatrix} \begin{bmatrix} T(t) \\ \mathbf{v}(t) \end{bmatrix} + \begin{bmatrix} \mathcal{B}_T & 0 \\ 0 & \mathcal{B}_{\mathbf{v}} \end{bmatrix} \begin{bmatrix} u_T(t) \\ u_{\mathbf{v}}(t) \end{bmatrix}$$

$$u_T(t) = H_T z_T(t) \qquad u_{\mathbf{v}}(t) = H_{\mathbf{v}} z_{\mathbf{v}}(t)$$

Actuator Dynamics

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$$u_T(t) = H_T z_T(t) \qquad u_{\mathbf{v}}(t) = H_{\mathbf{v}} z_{\mathbf{v}}(t)$$

$$(\Sigma_{aT}) \qquad \dot{z}_T(t) = \mathcal{A}_{aT} z_T(t) + \mathcal{B}_{aT} v_T(t)$$

$$(\Sigma_{a\mathbf{v}}) \qquad \dot{z}_{\mathbf{v}}(t) = \mathcal{A}_{a\mathbf{v}} z_{\mathbf{v}}(t) + \mathcal{B}_{a\mathbf{v}} v_{\mathbf{v}}(t)$$

Actuator Dynamics

$$(\Sigma) \qquad \frac{d}{dt}\begin{bmatrix} T(t) \\ \mathbf{v}(t) \end{bmatrix} = \begin{bmatrix} \mathcal{A}_T & \mathcal{F} \\ 0 & \mathcal{A}_{\mathbf{v}} \end{bmatrix}\begin{bmatrix} T(t) \\ \mathbf{v}(t) \end{bmatrix} + \begin{bmatrix} \mathcal{B}_T & 0 \\ 0 & \mathcal{B}_{\mathbf{v}} \end{bmatrix}\begin{bmatrix} u_T(t) \\ u_{\mathbf{v}}(t) \end{bmatrix}$$

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COMPOSITE SYSTEM

$$x(t) = \begin{bmatrix} T(t) & \mathbf{v}(t) & z_T(t) & z_{\mathbf{v}}(t) \end{bmatrix}^T$$

Composite System

$$(\Sigma_{full}) \qquad \dot{x}(t) = \tilde{\mathcal{A}}x(t) + \tilde{\mathcal{B}}\mathbf{v}(t) + \tilde{\mathcal{G}}w(t)$$

$$\tilde{A} = \begin{bmatrix} \mathcal{A}_T & \mathcal{F} & \mathcal{B}_T H_T & 0 \\ 0 & \mathcal{A}_{\mathbf{v}} & 0 & \mathcal{B}_{\mathbf{v}} H_{\mathbf{v}} \\ 0 & 0 & \mathcal{A}_{aT} & 0 \\ 0 & 0 & 0 & \mathcal{A}_{a\mathbf{v}} \end{bmatrix}$$

Composite System

$$(\Sigma_{full}) \qquad \dot{x}(t) = \tilde{\mathcal{A}}x(t) + \tilde{\mathcal{B}}\mathbf{v}(t) + \tilde{\mathcal{G}}w(t)$$

$$\tilde{A} = \begin{bmatrix} \mathcal{A}_T & \mathcal{F} & \mathcal{B}_T H_T & 0 \\ 0 & \mathcal{A}_{\mathbf{v}} & 0 & \mathcal{B}_{\mathbf{v}} H_{\mathbf{v}} \\ 0 & 0 & \mathcal{A}_{aT} & 0 \\ 0 & 0 & 0 & \mathcal{A}_{a\mathbf{v}} \end{bmatrix}$$

$$\tilde{\mathcal{B}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \mathcal{B}_{aT} & 0 \\ 0 & \mathcal{B}_{a\mathbf{v}} \end{bmatrix} \qquad \tilde{\mathcal{G}} = \begin{bmatrix} \mathcal{G}_T & 0 \\ 0 & \mathcal{G}_{\mathbf{v}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Composite System

$$(\Sigma_{full}) \qquad \dot{x}(t) = \tilde{\mathcal{A}}x(t) + \tilde{\mathcal{B}}\mathbf{v}(t) + \tilde{\mathcal{G}}w(t)$$

$$\tilde{A} = \begin{bmatrix} \mathcal{A}_T & \mathcal{F} & \mathcal{B}_T H_T & 0 \\ 0 & \mathcal{A}_\mathbf{v} & 0 & \mathcal{B}_\mathbf{v} H_\mathbf{v} \\ 0 & 0 & \mathcal{A}_{aT} & 0 \\ 0 & 0 & 0 & \mathcal{A}_{a\mathbf{v}} \end{bmatrix}$$

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Composite System

$$(\Sigma) \qquad \dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0 \in X$$

ADD "ACTUATOR" MODEL / DYNAMICS

$$(\Sigma_a) \qquad \dot{x}_a(t) = A_a x_a(t) + B_a v(t), \quad x_a(0) = x_{a0} \in X_a$$

$$u(t) = Hx_a(t)$$

Composite System

$$(\Sigma) \qquad \dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0 \in X$$

ADD "ACTUATOR" MODEL / DYNAMICS

$$(\Sigma_a) \qquad \dot{x}_a(t) = A_a x_a(t) + B_a v(t), \quad x_a(0) = x_{a0} \in X_a$$

$$u(t) = Hx_a(t)$$

COMPOSITE SYSTEM

$$\dot{x}(t) = Ax(t) + BHx_a(t), \quad x(0) = x_0 \in X$$

$$\dot{x}_a(t) = A_a x_a(t) + B_a v(t), \quad x_a(0) = x_{a0} \in X_a$$

Composite System

$$z(t) = \begin{bmatrix} x(t) & x_a(t) \end{bmatrix}^T \in Z \triangleq X \times X_a$$

$$(\Sigma_c) \qquad \dot{z}(t) = \tilde{A}z(t) + \tilde{B}v(t), \quad z(0) = z_0 \in Z$$

$$\tilde{A} = \begin{bmatrix} A & BH \\ 0 & A_a \end{bmatrix} \quad \tilde{B} = \begin{bmatrix} 0 \\ B_a \end{bmatrix}$$

Composite System

$$z(t) = \begin{bmatrix} x(t) & x_a(t) \end{bmatrix}^T \in Z \triangleq X \times X_a$$

$$(\Sigma_c) \qquad \dot{z}(t) = \tilde{A}z(t) + \tilde{B}v(t), \quad z(0) = z_0 \in Z$$

$$\tilde{A} = \begin{bmatrix} A & BH \\ 0 & A_a \end{bmatrix} \quad \tilde{B} = \begin{bmatrix} 0 \\ B_a \end{bmatrix}$$

ISSUES WITH COMPOSITE SYSTEMS

Composite System Example

$$(\Sigma) \qquad \dot{x}(t) = x(t) + u(t)$$

$$(\Sigma_a) \quad \dot{x}_a(t) = \frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v(t)$$

$$u(t) = Hx_a(t) = \begin{bmatrix} 1 & -1 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \qquad BH = 1 \cdot \begin{bmatrix} 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

Composite System Example

$$(\Sigma) \qquad \dot{x}(t) = x(t) + u(t)$$

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(Σ) and (Σ_a) are controllable but

Composite System Example

$$(\Sigma) \qquad \dot{x}(t) = x(t) + u(t)$$

$$(\Sigma_a) \quad \dot{x}_a(t) = \frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v(t)$$

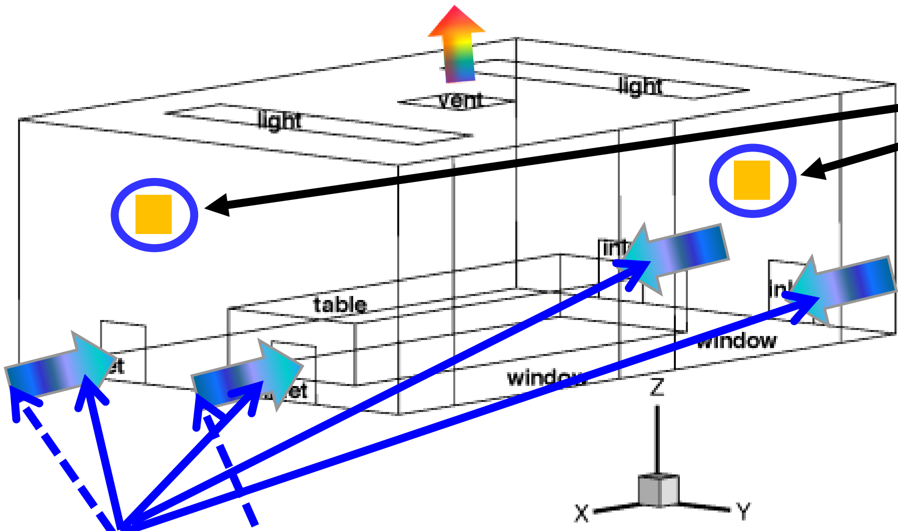
$$u(t) = Hx_a(t) = \begin{bmatrix} 1 & -1 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \qquad BH = 1 \cdot \begin{bmatrix} 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

(Σ) and (Σ_a) are controllable but

$$(\Sigma_c) \qquad \frac{d}{dt}\begin{bmatrix} x(t) \\ x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x(t) \\ x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_c(t)$$

(Σ_c) is not stabilizable

Boussinesq Equations



sensors / region

$$\Omega(\vec{\mathbf{q}}) = \left\{ \vec{\mathbf{x}} \in \bar{\Omega} : \left\| \vec{\mathbf{q}} - \vec{\mathbf{x}} \right\| < \delta \right\}$$

controlled region is around conference table

$u_T(t)$ = **inflow temperature**

$$T(t,\vec{\mathbf{x}})\,|_{\Gamma_c} = b_T(\vec{\mathbf{x}})u_T(t)$$

$$\frac{\partial}{\partial t}T(t,\vec{\mathbf{x}}) + \mathbf{v}(t,\vec{\mathbf{x}})\bullet\nabla T(t,\vec{\mathbf{x}}) = \kappa\nabla^2 T(t,\vec{\mathbf{x}}) + g_T(\vec{\mathbf{x}})w_T(t)$$

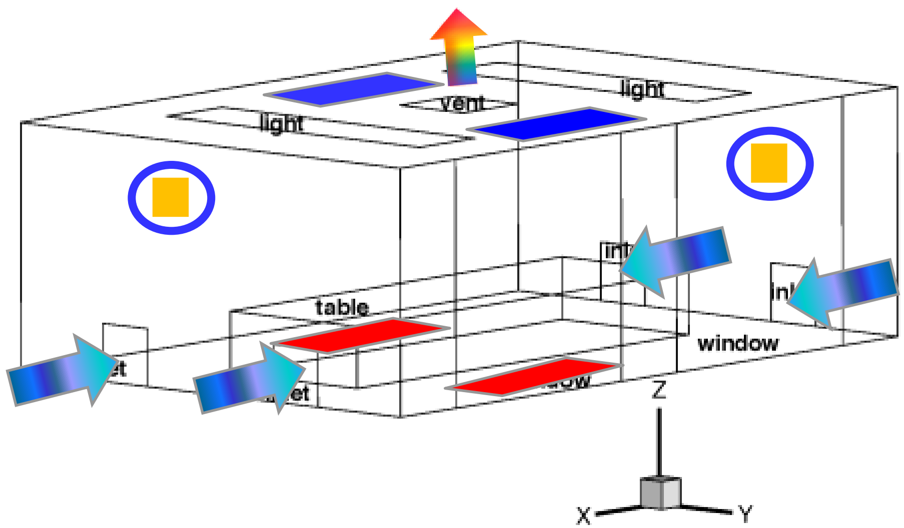
$$\frac{\partial}{\partial t}\mathbf{v}(t,\vec{\mathbf{x}}) + \left(\mathbf{v}(t,\vec{\mathbf{x}})\bullet\nabla\mathbf{v}(t,\vec{\mathbf{x}})\right) = \nu\nabla^2\mathbf{v}(t,\vec{\mathbf{x}}) - \nabla p(t,\vec{\mathbf{x}})$$

$$+ g\alpha_T e_d T(t,\vec{\mathbf{x}}) + g_{\mathbf{v}}(\vec{\mathbf{x}})w_{\mathbf{v}}(t)$$

$$\mathbf{v}(t,\vec{\mathbf{x}})\,|_{\Gamma_c} = b_{\mathbf{v}}(\vec{\mathbf{x}})u_{\mathbf{v}}(t)$$

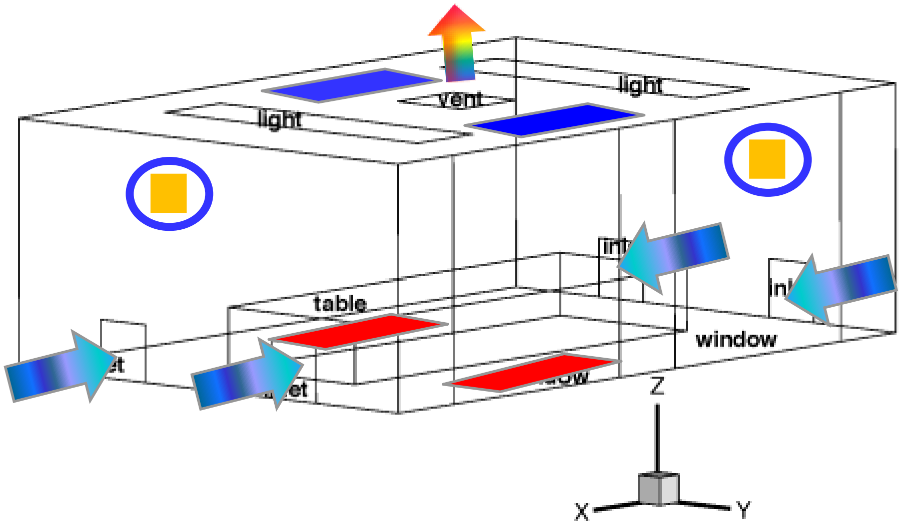
$$div\ \mathbf{v}(t,\vec{\mathbf{x}}) = 0$$

Building DPS Control ProblemS



- MODEL REDUCTION
- MODEL IDENTIFICATION
- PDE OPTIMIZATION
- PROBLEM SELECTION
- SENSOR LOCATION
- ACTUATOR LOCATION
- ACTUATOR DYNAMICS
- <u>CONTROLLER DESIGN</u>
  - ROBUSTNESS
  - UNCERTAINTY
  - MULTI-SCALE
- ...

Building DPS Control ProblemS

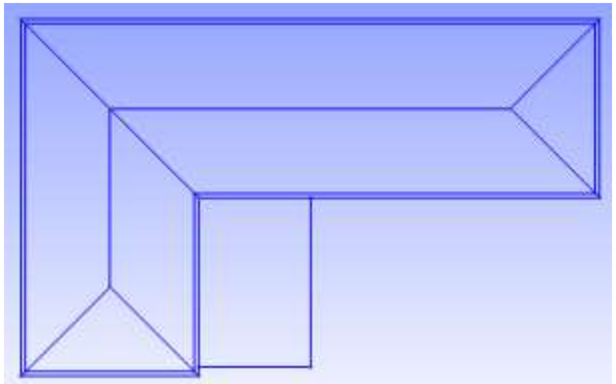


COMPUTATIONAL ISSUES

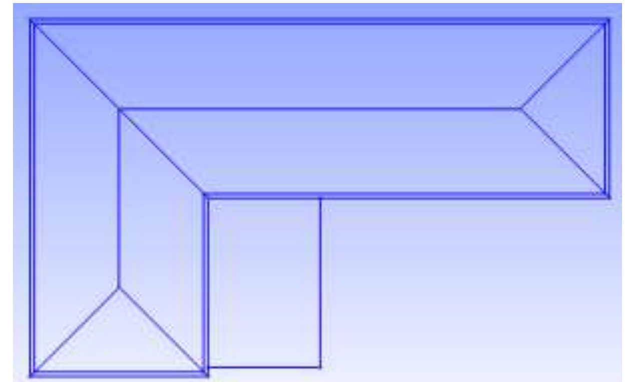
- MODEL REDUCTION
- CONTROLLER REDUCTION
- SIMULATION
  - OPEN LOOP
  - CLOSED LOOP

- MODEL REDUCTION
- MODEL IDENTIFICATION
- PDE OPTIMIZATION
- PROBLEM SELECTION
- SENSOR LOCATION
- ACTUATOR LOCATION
- ACTUATOR DYNAMICS
- CONTROLLER DESIGN
  - ROBUSTNESS
  - UNCERTAINTY
  - MULTI-SCALE
- …

What About REAL Buildings?

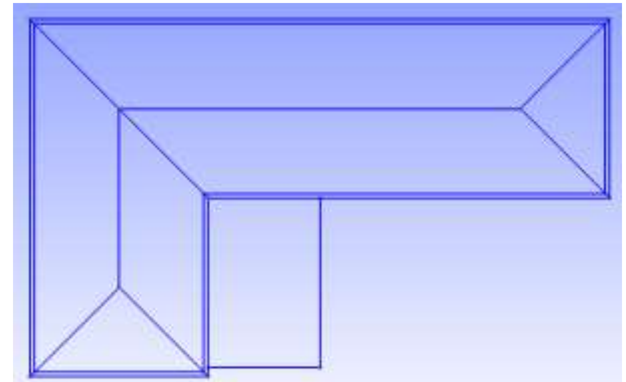


What About REAL Buildings?

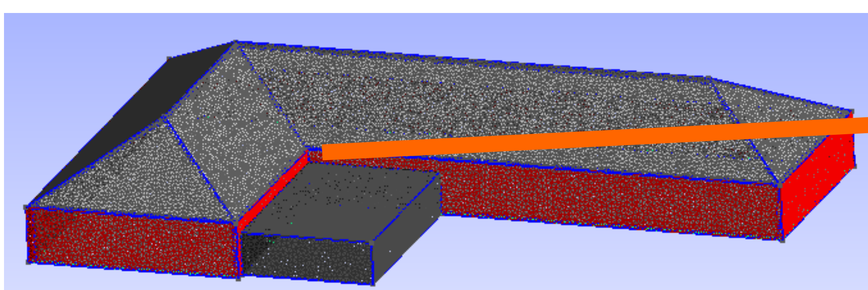


PRACTICAL SIMULATION, DESIGN & CONTROL <u>TOOLS</u>

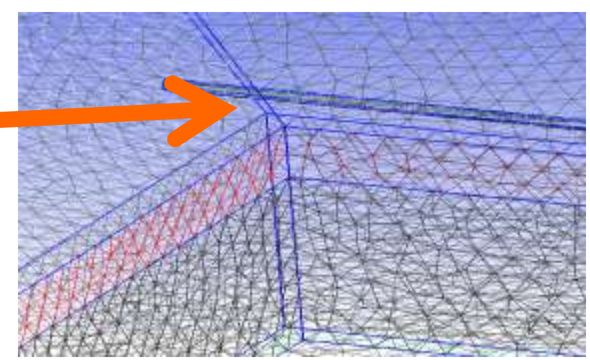
What About REAL Buildings?



PRACTICAL SIMULATION, DESIGN & CONTROL TOOLS



MESH



ZOOM MESH

HP²C Enabled Model Reduction

Cluster, Grid, HPC

High Performance
High Productivity
Computing

High Fidelity Physics-Based Simulation

$$\begin{cases} \partial_t \theta + u \cdot \nabla \theta = \kappa \Delta \theta \\ \partial_t u + u \cdot \nabla u + \nabla p = \nu \Delta u + \beta \theta e_3 \\ \nabla \cdot u = 0 \\ u|_{t=0} = u_0, \quad \theta|_{t=0} = \theta_0. \end{cases}$$

ADVANCED MODEL REDUCTION METHODS

HP²C ENABLED MODELING & COMPUTER DESIGN TOOLS

BUILDING **PHYSICS** DESIGN PARAMETERS

Hierarchal Reduced – Order Design & Control Models

$$\dot{x}_n = f_n(x_n, u, q) + v$$

INTERACTIVE

INTEROPERABLE

COMPUTER DESIGN TOOL

Building Description

- Architecture, Materials
- HVAC, Loads, Lights

…

- Occupancy
- Sensors, Actuators …

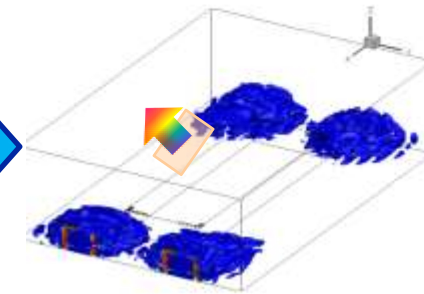
HP²C Enabled Controller Reduction

High Fidelity Physics-Based Control Design Tools

$$\begin{cases} \partial_t\theta + u\cdot\nabla\theta = \kappa\Delta\theta \\ \partial_t u + u\cdot\nabla u + \nabla p = \nu\Delta u + \beta\theta e_3 \\ \nabla\cdot u = 0 \\ u|_{t=0} = u_0, \quad \theta|_{t=0} = \theta_0. \end{cases}$$

DISTRIBUTED PARAMETER CONTROL

HP²C



$$u(t) = -\iiint_{\Omega} k_T(\vec{\mathbf{x}})\theta(t,\vec{\mathbf{x}})d\vec{\mathbf{x}}$$

BUILDING PHYSICS DESIGN PARAMETERS

ADVANCED CONTROLLER REDUCTION METHODS

Building Description

- Architecture, Materials
- HVAC, Loads, Lights

…

- Occupancy
- Sensors, Actuators …

NEW INFORMATION ABOUT

- WHAT MUST BE SENSED
  OR
- WHAT MUST BE ESTIMATED
- WHERE TO PLACE SENSORS "VENTS"
- ROBUSTNESS
- SENSITIVITY
- …

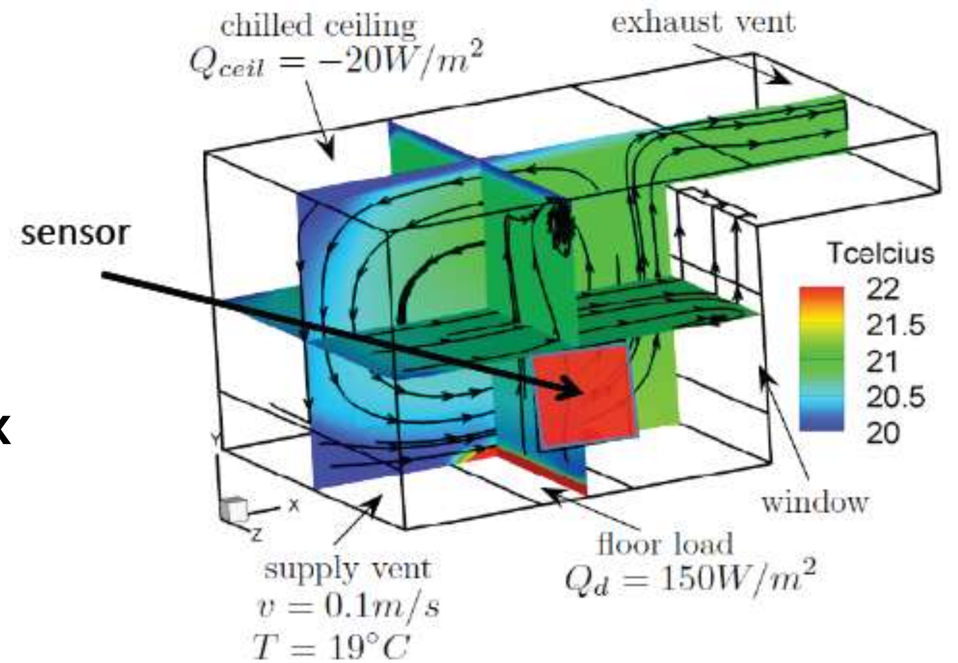
Holistic Reduced Order Controller

$$\dot{z}_e^h(t) = A_e^h z_e^h(t) + F_e^h(\mathbf{q})[y(t) - C_e^h(\mathbf{q})z_e^h(t)]$$

$$u(t) = -\iiint_{\Omega} k_T(\vec{\mathbf{x}})\theta_e^h(t,\vec{\mathbf{x}})d\vec{\mathbf{x}}$$

Disturbance Rejection: ICAM CUBE

Small room (4m x 4m x 3m) with displacement ventilation and chilled ceiling



Inputs

- **Control: chilled ceiling heat flux**
- **Disturbance: floor heat flux**

Outputs

- **Temperature measurements, on two walls parallel to the XY planes**
- **Occupied zone averaged temperature (used to define LQR cost)**

Boussinesq equations; k-ε model

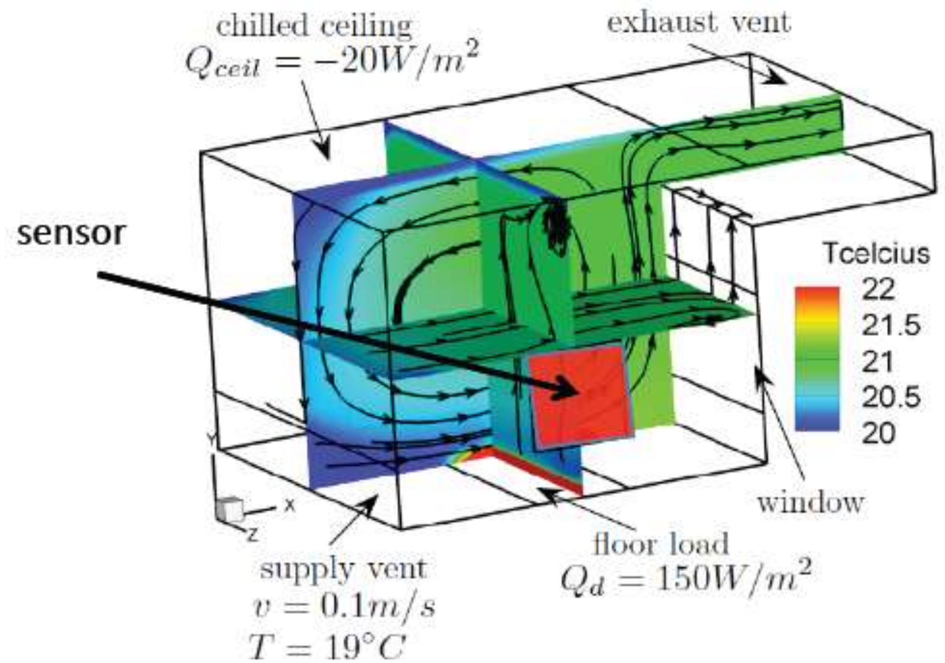
- **Grid: 6.6 x 10^4 nodes**
- **Re = 2.2x10^4 (w.r.t. room h=3m, diffuser vel.=0.1m/s)**
- **Gr = 4.7x10^9 (w.r.t. room h=8m)**
- **Re^2/Gr = 0.1 < 1 => buoyancy dominated**
- **Time-step = 2 sec**

Disturbance Rejection: ICAM CUBE

$$\dot{z}(t) = A_r z(t) + B_r u(t) + G_r v(t)$$

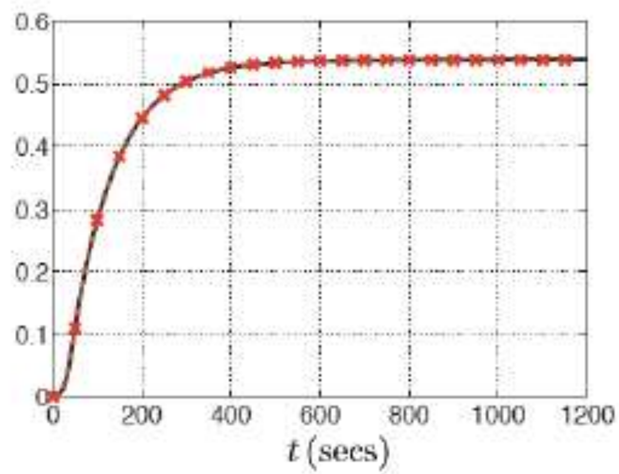
$$\xi(t) = D z_r(t)$$

$$y(t) = C(q) z(t) + E w(t)$$

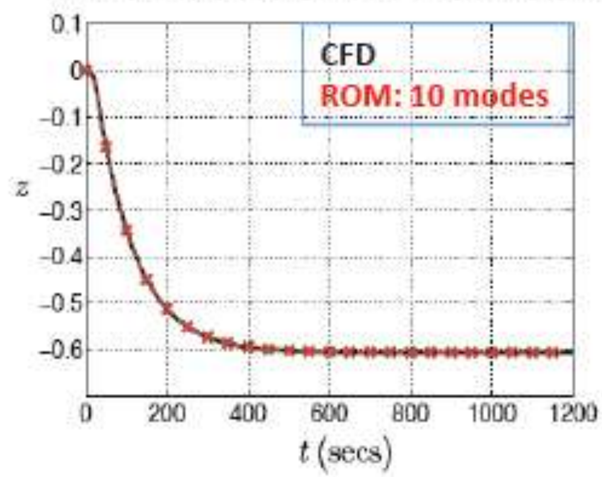


Perturbations in average occupied zone temperature

Step the floor flux from 150 to 190 W/m²

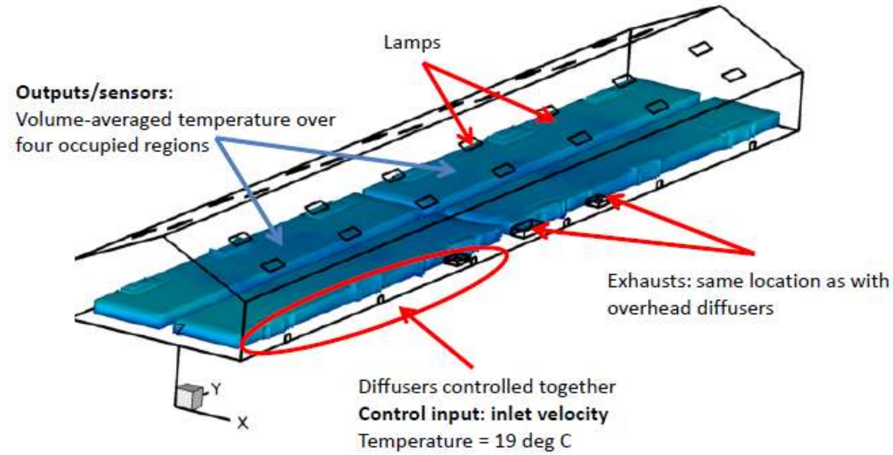


Step the ceiling flux from -20 to -30 W/m²



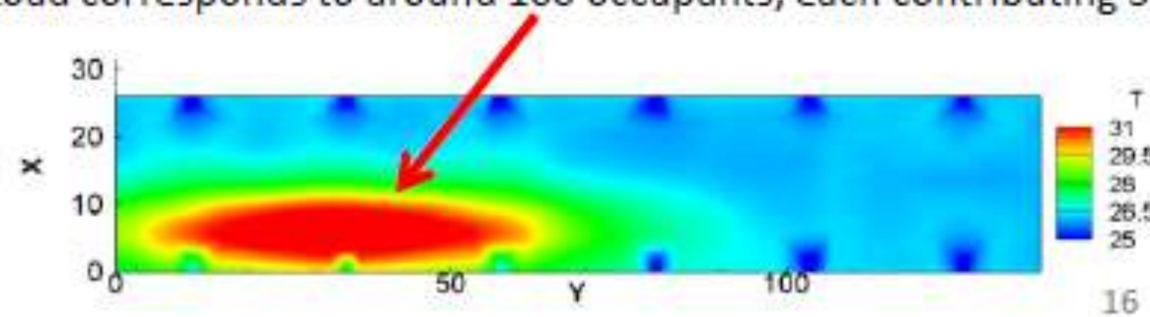
Results for a Real Building

Low-energy system: displacement ventilation

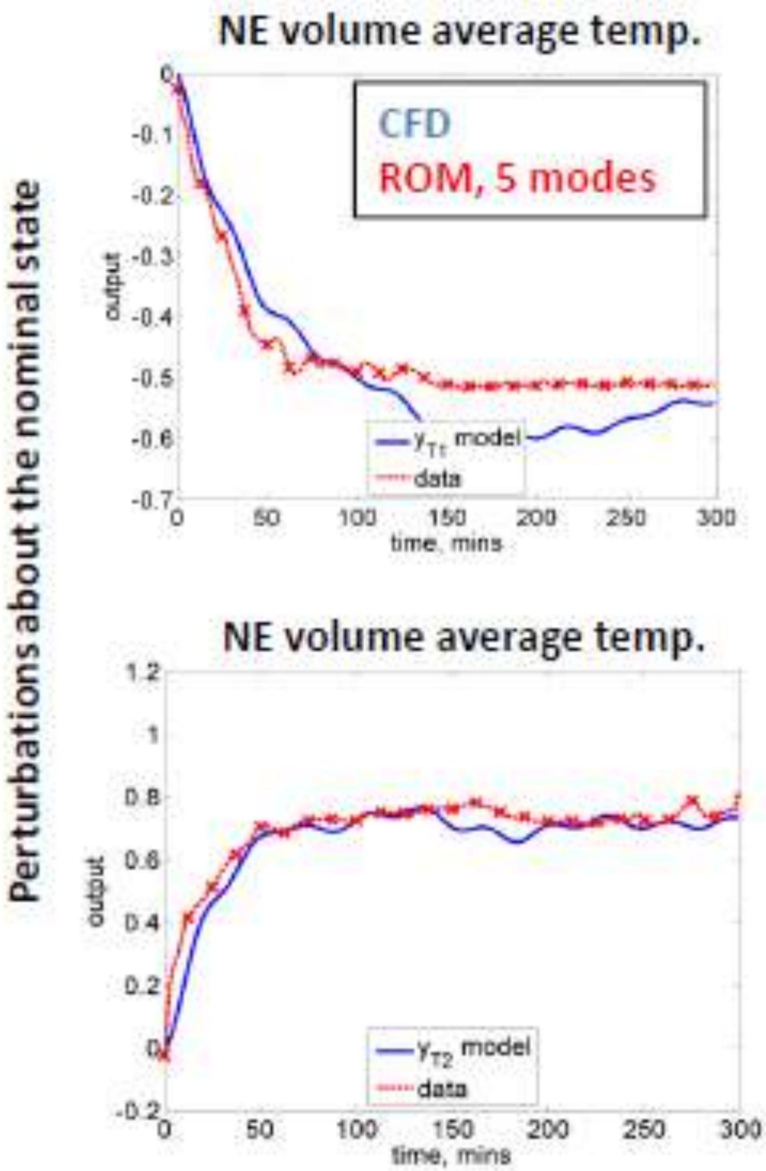


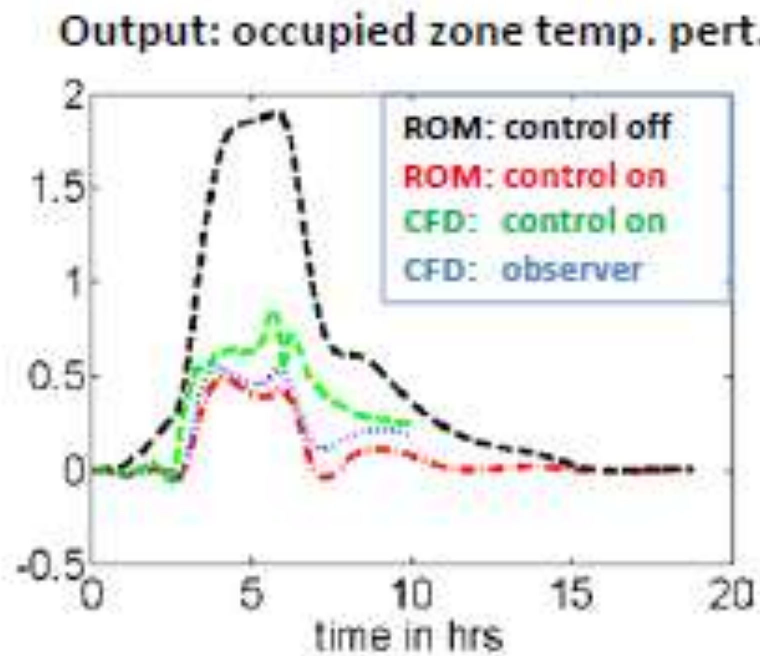
- **Re = 1.2×10^5(w.r.t. room h=8m, diff. v=0.2m/s)**
  - **Gr = 9×10^{10} (w.r.t. room h=8m)**
    - **Re2/Gr = 0.16 < 1**

- Floor load (occupants and solar) modeled as a Gaussian
- Floor flux varied from 0 to 8.5 W/m^2 (normalized w.r.t. floor area)
- Load corresponds to around 100 occupants, each contributing 300 W



Results for a Real Building

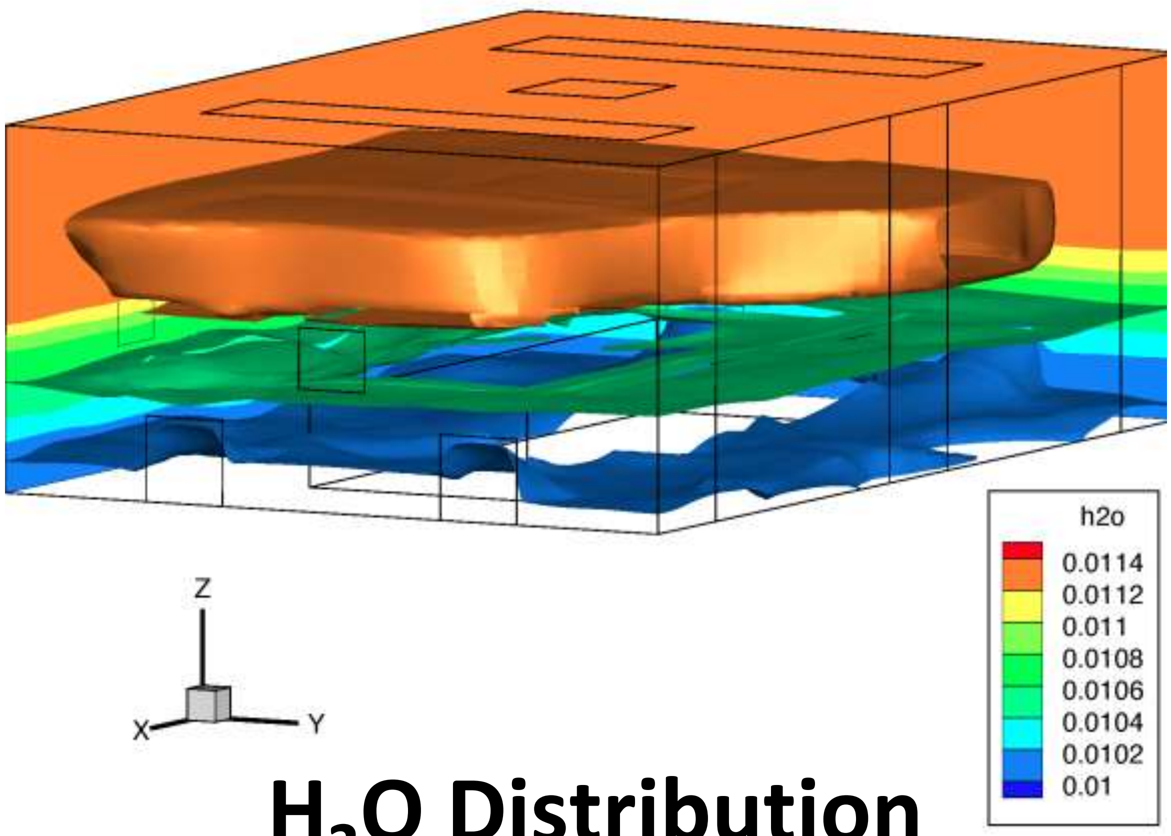




(MW-hr)	TrnSys	CFD-ROM	% savings
Supply fan power	7.9	6.4	19.0
Return fan power	2.8	2.2	21.4
Chiller power	26.0	10.7	58.8

More Complex Physics

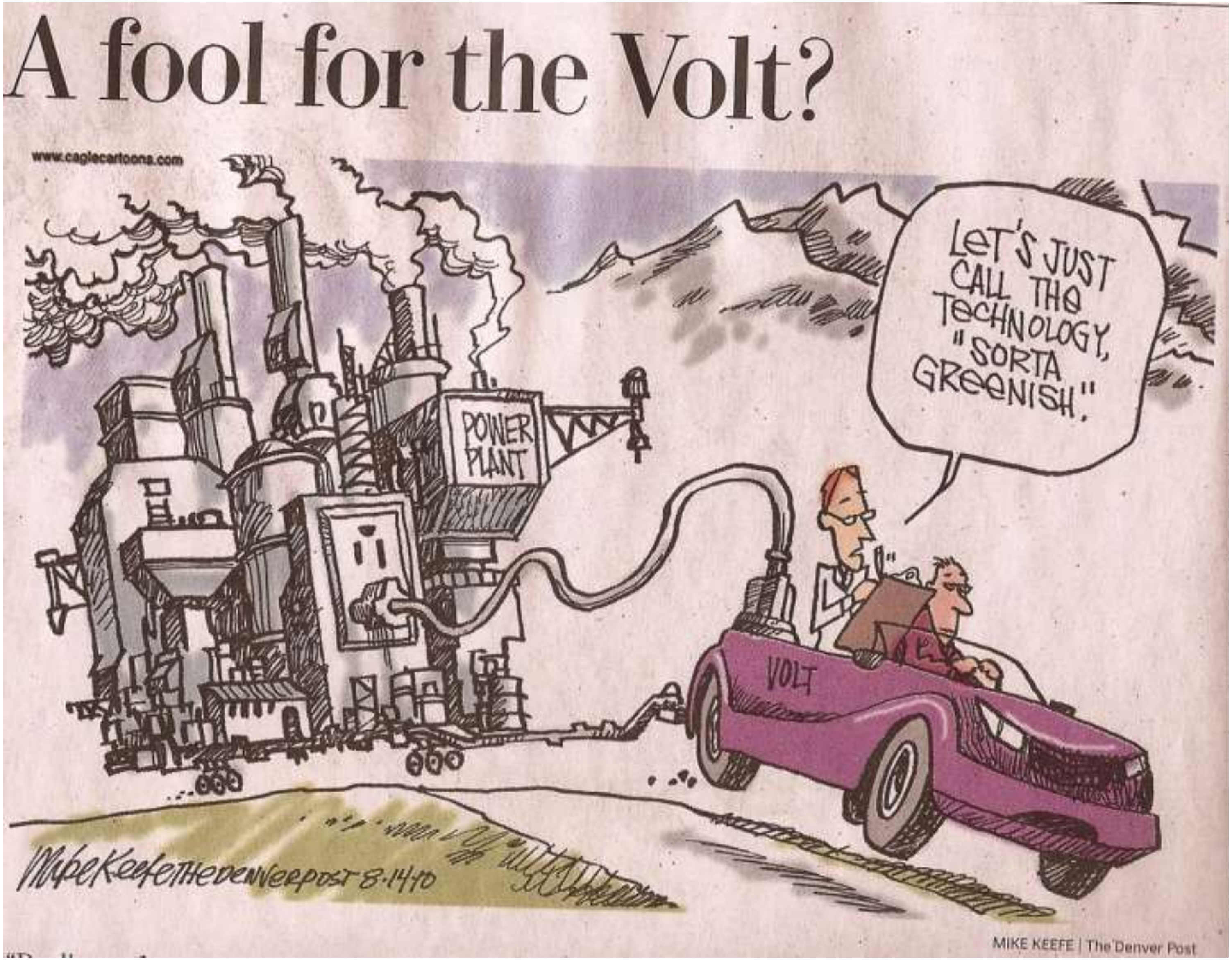
- **3 CONTROLLERS; 1 TEMP SENSOR; 2 CONTROLLED OUTPUTS …**
- **12th ORDER H^2 CONTROLLER …**



H_2O Distribution

There is no Free Lunch (Energy)

There is no Free Lunch (Energy)



THANK YOU

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Jaburns@vt.edu