ELECTROMAGNETIC PURSUIT-EVANSION PROBLEMS, RELAXED CONTROLS AND PREISACH HYSTERESIS

H. T. BANKS

Center for Research in Scientific Computation

Center for Quantitative Sciences in Biomedicine

NC STATE University

Raleigh, N. C.

In Honor of Len Berkovitz
Control of DPS-Wuppertal
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CRSC
Center for Research in Scientific Computation

Center for Quantitative Sciences in Biomedicine
North Carolina State University

Static and Dynamic Two-Player Evasion-Pursuit Non-cooperative Games with Uncertainty

- *Computational efforts on electromagnetic interrogation methodology (inverse scattering problems)*

- *Counter-interrogation and counter-counter-interrogation methodology (evader/interrogator capabilities)*

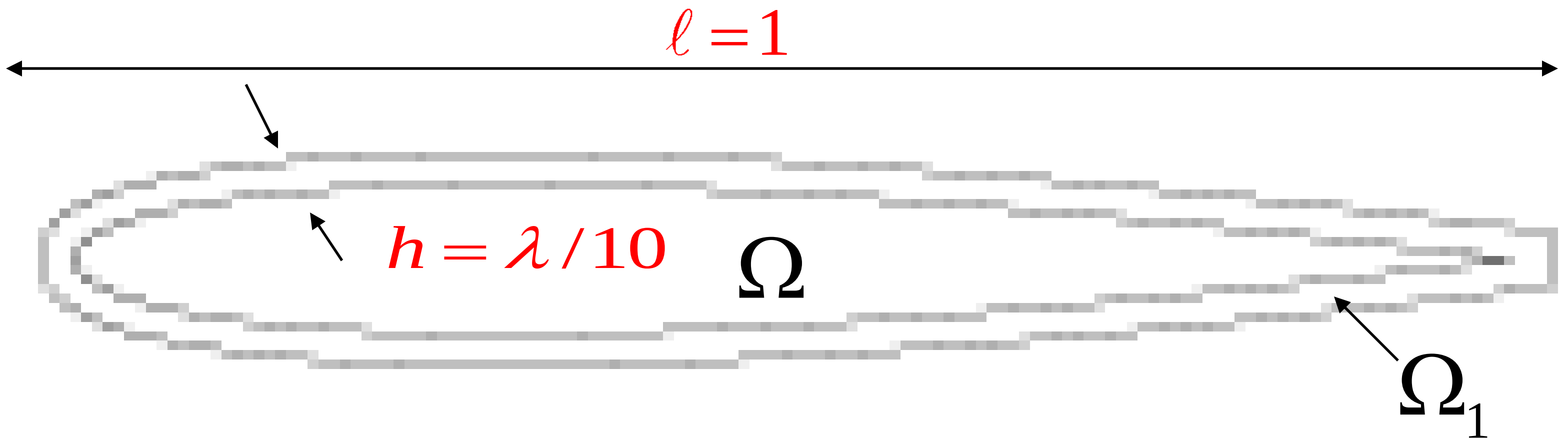
- *Formulation as a static two-player evasion-pursuit non-cooperative games with uncertainty (a probability based inverse problem framework -- Prohorov Metric Framework)*

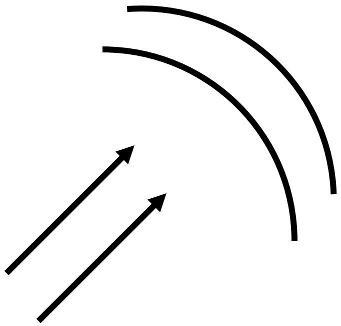
- *Dynamic evasion-pursuit in a Markov diffusion process/semigroup control framework*

- *Relaxed controllers, Preisach hysteresis, mixing distributions in statistical inverse problems*

II. REDUCED RCS

2-D SECTION OF AN AIRFOIL





far field $F_\alpha(\theta)$

$$E_x^{in} = e^{i(k_y y + k_z z)}$$

$$k_y = k \cos \alpha,$$

$$k_z = k \sin \alpha$$

interrogating plane waves with wavelength λ *: return* F_α *minimized over* $\alpha \in A = [\alpha_0, \alpha_1] = [0, \pi]$

Two Dimensional Problem

Computations for time-harmonic, transverse magnetic (TM_x) mode: Maxwell's reduces to Helmholtz for $E_x = E_x^{(r)} + E_x^{(i)}$

$$\nabla \cdot \left(\frac{1}{\mu} \nabla \frac{\partial E_x^{(r)}}{\partial n}\right) + \varepsilon \omega^2 E_x^{(r)} = -\nabla \cdot \left(\frac{1}{\mu} \nabla E_x^{(i)}\right) + \varepsilon \omega^2 E_x^{(i)} \quad \text{in } \mathrm{R}^2 \setminus \overline{\Omega}$$

$$E_x = E_x^{(r)} + E_x^{(i)} = 0 \qquad \text{on } \partial\Omega$$

$$\left[\frac{1}{\mu} \frac{\partial E_x}{\partial n}\right]_-^+ = [\, E_x \,]_-^+ = 0 \qquad \text{on } \partial\Omega_1 \setminus \partial\Omega$$

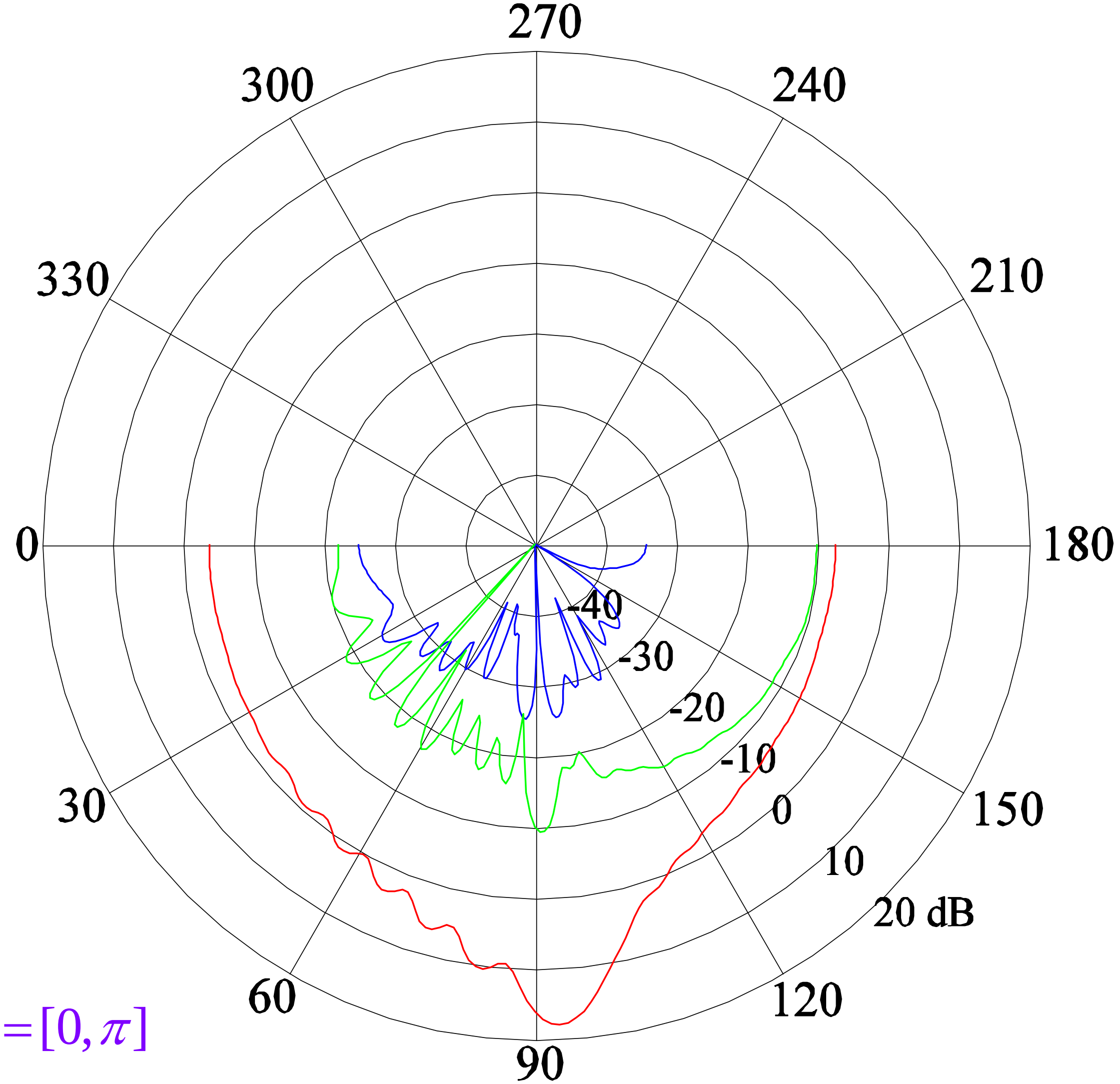
$$\lim_{r \to \infty} \sqrt{r} \left(\frac{\partial E_x^{(r)}}{\partial r} - ikE_x^{(r)}\right) = 0 \quad \text{(Silver-Müller/Sommerfeld)}$$

RADAR CROSS SECTION OF AN AIRFOIL

No coating:

Optimization of complex dielectric permittivity ε_r

Complete optimization (ε_r, μ_r)

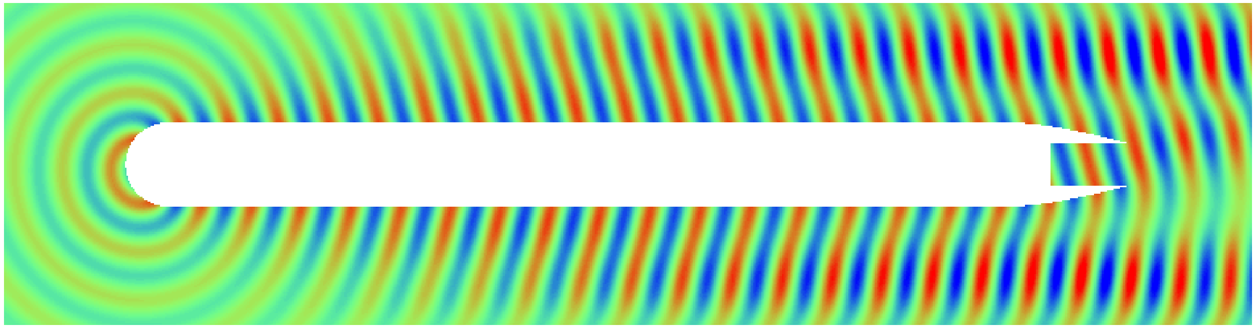


$\alpha \in A = [\alpha_0, \alpha_1] = [0, \pi]$

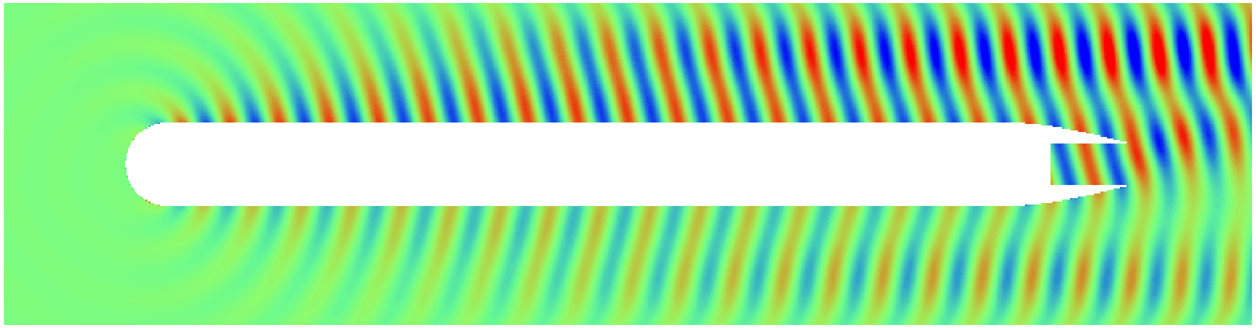
$$RCS(\alpha) = 10\log_{10}\left(\frac{1}{8\pi}\left|F_\alpha(\alpha+\pi)\right|^2\right) \qquad \lambda = \frac{1}{4}$$

REFLECTED FIELD FOR UNCOATED "CRUISE MISSILE"

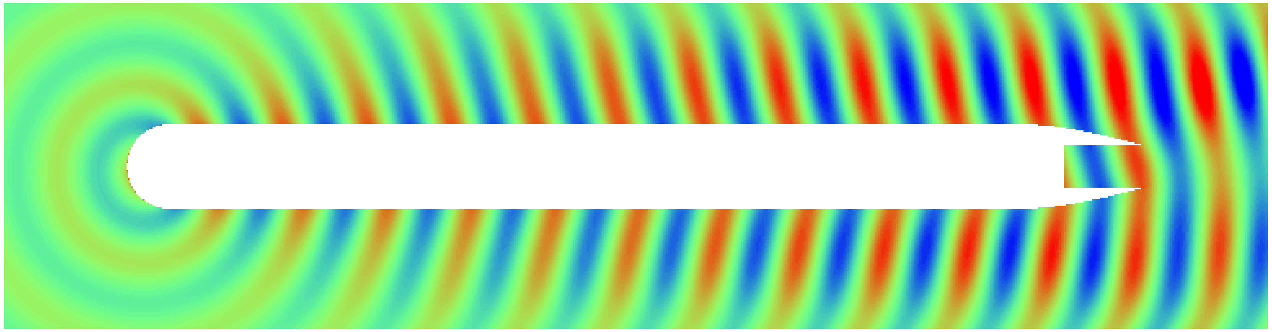
$\alpha = 15°$



REFLECTED FIELD FOR COATED "CRUISE MISSILE" USING DESIGN INTERROGATING FREQUENCY f=1 GHz



REFLECTED FIELD FOR COATED "CRUISE MISSILE" USING INTERROGATING FREQUENCY f=.6GHz



CONCLUSIONS I:

WITH INFORMATION ON INTERROGATING FREQUENCIES, <u>EVADER</u> CAN DESIGN TO <u>WIN</u>; LOW TECH USE OF COUNTER INTERROGATION OR INFO ON DESIGN OF COATINGS IS READILY OVERCOME-- <u>INTERROGATOR WINS</u>!

DESIGN AND INTERROGATION BASED ON PLANAR REFECTION COEFFICIENTS MAY BE ADEQUATE (AS OPPOSED TO MUCH MORE COMPUTATIONALLY INTENSIVE FAR FIELD FEM FORMULATION) FOR REAL-TIME COUNTER-COUNTER-INTERROGATION

CONCLUSIONS II:

Evader and interrogator must each try to confuse the other by introducing <u>uncertainty</u> in their design and interrogating strategies; several approaches

1. "mixed strategies" — two player static games with probabilistic strategy formulations

2. formulation of reflection intensities as Markov process with controlled dynamics

Counter- and Counter-counter Interrogation as Static Zero-Sum Two Player Game (evader-interrogator)

$Evader: uses\ design\ parameters\ (\varepsilon, \mu) \in \mathcal{E} \times \mathcal{M}$

$Interrogator: uses\ angles\ of\ incidence\ \varphi \in \Phi,\ interrogating\ frequencies\ f \in \mathcal{F}$

$Probability\ distributions: P_e(\varepsilon, \mu) = P_e^1(\varepsilon) P_e^2(\mu)\ over\ \mathcal{E} \times \mathcal{M}$

$P_i(\varphi, f) = P_i^1(\varphi) P_i^2(f)\ over\ \Phi \times \mathcal{F}$

$For\ P_e \in \mathcal{P}(\mathcal{E} \times \mathcal{M}), P_i \in \mathcal{P}(\Phi \times \mathcal{F}),\ define$

$$J(P_e, P_i) = \int_{\mathcal{E} \times \mathcal{M}} \int_{\Phi \times \mathcal{F}} \left| R(\varepsilon, \mu, \varphi, f) \right|^2 dP_e(\varepsilon, \mu)\, dP_i(\varphi, f)$$

For general min–max *problem, evader optimizes over* $P_e \in \mathscr{P}(\mathcal{E} \times \mathcal{M})$, *while interrogator optimizes over* $P_i \in \mathscr{P}(\Phi \times \mathscr{F})$ *where* $\mathscr{P}$ *is metric space of probability measures with the Prohorov metric (the metric of weak convergence of probability measures, i.e., weak** *in* $\mathscr{P} \subset C^*$) –

Theoretical and computational <u>*inverse problem*</u> *framework*:

(*compactness, stability, approximation with Dirac measures, splines, etc*) *in* [*Banks & Bihari, Inverse Problems*, 17 (2001), 95–111],[*Banks & Pinter, SIAM J. Multiscale Modeling and Simulation*, 3 (2005), 395–412]. *Counter interrogation and counter–counter interrogation computations in* [*Banks, Kepler, Ito & Toivanen, SIAM J. Appl. Math*, 66 (2006)] [*Banks, Ito & Toivanen, Comm. in Comp. Physics*, 1 (2006)]

RANDOM VARIABLES and ASSOCIATED METRIC SPACES

$\mathcal{P} = \mathcal{P}(\mathrm{Q}) = \{\, P \;:\; P \text{ are probability measures on } Q \,\}.$

$(P(Q), \rho)$ *is a metric space with the Prohorov metric* ρ.

It is a complete metric space and is compact if Q *is compact.*

PROHOROV METRIC (weak* convergence for $\mathcal{P}(\mathrm{Q}) \subset \mathrm{C}^*(\mathrm{Q})$)

$$\rho(P^k, P) \to 0 \;\Leftrightarrow\; \int_Q g\,dP^k \to \int_Q g\,dP \quad \text{for all } g \in C(Q)$$

$$\Leftrightarrow \text{ convergence in expectation}$$

$$\Leftrightarrow\; P^k[A] \to P[A] \;\text{ for all Borel } A \subset Q \text{ with } P(\partial A) = 0$$

For details on Prohorov metric and an approximation theory, see[1].

[1] H.T.Banks and K.L.Bihari, Modeling and estimating uncertainty in parameter estimation, CRSC-TR99-40, NCSU, Dec.,1999; Inverse Problems 17(2001),1-17.

Initial efforts on static two – player evasion – interrogation games with uncertainty in

[*HTB*, <u>*Grove*</u>, *Ito* & *Toivanen*, *CRSC* – *TR*06 – 16, *June* 2006; *Comp*. & *Appl*. *Math*, **25** (2006), 289--306]

Game theory : MinMax problem (non – cooperative)

Find $P_e^* \in \mathscr{P}(\mathcal{E}\times\mathcal{M}), P_i^* \in \mathscr{P}(\Phi\times\mathcal{F})$ *such that*

$$J(P_e^*, P_i^*) = \min_{\mathscr{P}(\mathcal{E}\times\mathcal{M})} \max_{\mathscr{P}(\Phi\times\mathcal{F})} J(P_e, P_i)$$

Upper and lower values of game : $(\bar{J} \geq \underline{J})$

$$\bar{J} = \inf_{\mathscr{P}(\mathcal{E}\times\mathcal{M})} \sup_{\mathscr{P}(\Phi\times\mathcal{F})} J(P_e, P_i) \quad (\textit{security level for evader})$$

$$\underline{J} = \sup_{\mathscr{P}(\Phi\times\mathcal{F})} \inf_{\mathscr{P}(\mathcal{E}\times\mathcal{M})} J(P_e, P_i) \quad (\textit{security level for interrogator})$$

if $J^* = \bar{J} = \underline{J}$, *then* J^* *called* **value** *of the game and if there exists* (P_e^*, P_i^*) *such that* $J^* = J(P_e^*, P_i^*)$, *then* (P_e^*, P_i^*) *is called* **saddle point solution** *or* **non-cooperative equilibrium** *of the game*

Theorem: (*Von Neumann*)

Suppose X_0, Y_0 *are compact, convex subsets of metric linear spaces* X, Y *respectively. Further suppose that*

(*i*) *for all* $y \in Y_0$, $x \to f(x, y)$ *is convex and lower semi-continuous;*

(*ii*) *for all* $x \in X_0$, $y \to f(x, y)$ *is concave and upper semi-continuous.*

Then there exists a saddle point (x^*, y^*) *such that*

$$f(x^*, y^*) = \min_{X_0} \max_{Y_0} f(x, y)$$

Theorem:

Suppose $\mathcal{E},\mathcal{M},\Phi,\mathcal{F}$ *are compact and the spaces* $X_0=\mathcal{P}(\mathcal{E}\times\mathcal{M}), Y_0=\mathcal{P}(\Phi\times\mathcal{F})$ *are taken with the* *Prohorov metric*. *Then* X_0, Y_0 *are compact, convex subsets of* $X=C_B^*(\mathcal{E}\times\mathcal{M})$ *and* $Y=C_B^*(\Phi\times\mathcal{F})$, *respectively.*

Moreover, there exists $(P_e^*,P_i^*)\in\mathcal{P}(\mathcal{E}\times\mathcal{M})\times\mathcal{P}(\Phi\times\mathcal{F})$ *such that*

$$J(P_e^*,P_i^*)=\min_{\mathcal{P}(\mathcal{E}\times\mathcal{M})}\max_{\mathcal{P}(\Phi\times\mathcal{F})}J(P_e,P_i).$$

For computations, $dP_e(\varepsilon,\mu) \approx \sum p_e^j \delta_{(\varepsilon_j,\mu_j)}(\varepsilon,\mu)\,d\varepsilon\,d\mu,$ $dP_i(\varphi, f) \approx \sum p_i^j \delta_{(\varphi_j, f_j)}(\varphi, f)\,d\varphi\,df.$ *Convergence theory in Banks – Bihari,* 2001.

Correspond to Von Neumann's **finite mixed strategies** *framework for protection by disguising intentions from opponents – i.e., introduce uncertainty in choices of players*

DYNAMIC EVASION-INTERROGATION GAMES

H.T. B., S. Hu, K.Ito and S. Grove Muccio, Dynamic electromagnetic evasion-pursuit games with uncertainty, CRSC TR10-13, NCSU, August, 2010; Numerical Mathematics: Theory, Methods and Applications, 4 (2011), 399--418.

Formulation using Markov Diffusion Dynamics with Control

Evader's viewpoint: *Design best strategy for material dielectric (dynamics involve control of* $\varepsilon = \varepsilon(t)$*) – uncertainty in interrogating frequency* ω

Quantity of interest: $X(t,\varepsilon,\omega)$ = *intensity of reflected field -- e.g., function of reflection coefficients (Fresnel)* $\mathrm{r}(\varepsilon,\omega)$

Uncertainty: *i) in interrogating frequency* ω *is random variable – given by Markov diffusion process*

ii) in evader control of dielectric permittivity ε

Interrogating frequency dynamics:

$$d\omega_t = \mu(\omega_t)dt + \sigma(\omega_t)dB_t$$

Two formulations:

I. Evolution of Probability Density of Function of Intensity—
leads to controlled Fokker-Planck or forward Kolmogorov equation for probability density ϱ of measure of intensity:

$$\frac{\partial \varrho(t,\omega)}{\partial t} + \frac{\partial}{\partial \omega}\left[\mu(\omega)\varrho(t,\omega)\right] = \frac{1}{2}\frac{\partial^2}{\partial \omega^2}\left[\sigma^2(\omega)\varrho(t,\omega)\right] - \lambda(\varrho(t,\omega) - u(t,\omega))$$

II. Evolution of Expected Value of Intensity—
leads to controlled backward Kolmogorov equation for expected value v of measure of reflected intensity:

$$\frac{\partial}{\partial t}v(t,\omega) = \mu(\omega)\frac{\partial}{\partial \omega}v(t,\omega) + \frac{1}{2}\sigma^2(\omega)\frac{\partial^2}{\partial \omega^2}v(t,\omega) + \lambda(u(t,\omega) - v(t,\omega))$$

Control Dynamics

Control approximations (as in two sided static games)

$$U(t,\omega) = \int_{\mathcal{E}} r(\omega,\varepsilon)\, d\mathcal{U}(t,\varepsilon) \approx \int_{\mathcal{E}} r(\omega,\varepsilon) \sum_{j=1}^{M} \varepsilon_j(t)\, \delta_{\varepsilon_j^*}(\varepsilon)\, d\varepsilon$$

$$= \sum_{j=1}^{M} \varepsilon_j(t)\, u(\omega,\varepsilon_j^*) = \sum_{j=1}^{M} \varepsilon_j(t)\, b_j(\omega) = B(\omega)\,\overline{\varepsilon}(t)$$

$$\text{where } b_j(\omega) = R(\varepsilon_j^*,\omega),\ B(\omega) = (b_1(\omega),\ldots,b_M(\omega)),$$

$$\overline{\varepsilon}(t) = (\varepsilon_1(t),\ldots,\varepsilon_M(t))^T$$

Control Problem (LQR *problem*)

$$J(\varepsilon) = \int_0^\infty \int_{\underline{\omega}}^{\bar{\omega}} \left| v(\omega, t) \right|^2 d\omega dt + \int_0^\infty \beta \left| \bar{\varepsilon}(t) \right|^2 dt$$

subject to

$$\frac{\partial v}{\partial t} = Av + \lambda B\bar{\varepsilon} \quad \textit{with} \quad A\varphi = \eta \frac{\partial^2 \varphi}{\partial \omega^2} + \mu \frac{\partial \varphi}{\partial \omega} - \lambda \varphi$$

Can apply abstract LQR theory in Hilbert spaces

$$\mathcal{H} = \mathrm{L}^2(\underline{\omega}, \bar{\omega}), \quad \mathcal{V} = \mathrm{H}^1(\underline{\omega}, \bar{\omega})$$

(*see summary in Chapter* 7,[*HTB, R. Smith, Y. Wang, Smart Material Structures* : *Modeling, Estimation and Control, Wiley / Masson*, 1996])

Optimal Feedback or Adaptive Control

$$\bar{\varepsilon}^*(t) = -\frac{1}{\beta} B^* \Pi v(t) \quad \textit{where } B^* : \mathcal{H} \to \mathrm{R}^M,\ A\!: \mathcal{D}(A) \subset \mathcal{V} \to \mathcal{V}^*,$$

and $\Pi \in \mathcal{L}(\mathcal{V}^*, \mathcal{V})$ *satisfies an operator Riccati equation*

$$(A^*\Pi + \Pi A - \Pi B \beta^{-1} B^* \Pi + I) z = 0 \quad \textit{for all } z \in \mathcal{V},$$

and

$$A - \lambda B \beta^{-1} B^* \Pi$$

is the closed loop infinitesimal generator of an exponentially stable C_0 *semigroup---leads to efficient theoretical and computational framework!*

Extensive theoretical and computational tools:

1) $A = \eta \frac{\partial^2}{\partial \omega^2} + \frac{\partial}{\partial \omega} - \lambda$ *is infinitesimal generator of*

C_0 *semigroup* $S(t)$ *and relationship to Markov process* $X(t)$:

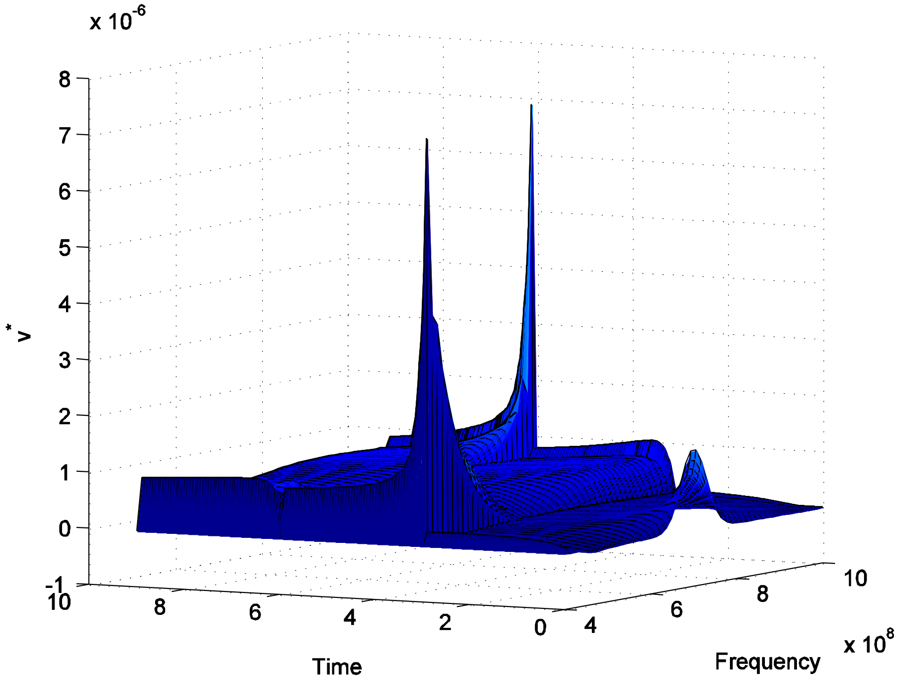
S.N.Ethier and T.G.Kurtz, Markov Processes: Characterization and Convergence, 1986

2) *Can use approximate LQR theory (with computational methodology) in Hilbert spaces*

$$\mathcal{H} = L^2(\underline{\omega}, \overline{\omega}), \quad \mathcal{V} = H^1(\underline{\omega}, \overline{\omega})$$

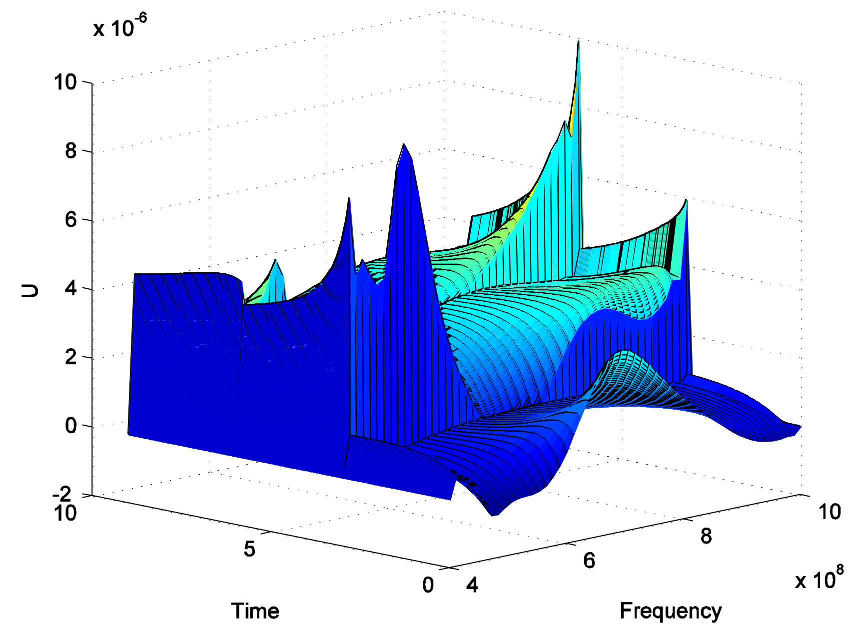
(*again see summary in Chapter* 7, [*HTB, R. Smith, Y. Wang, Smart Material Structures: Modeling, Estimation and Control, Wiley / Masson*, 1996])

Varying strategies: Normal-Gamma-Beta

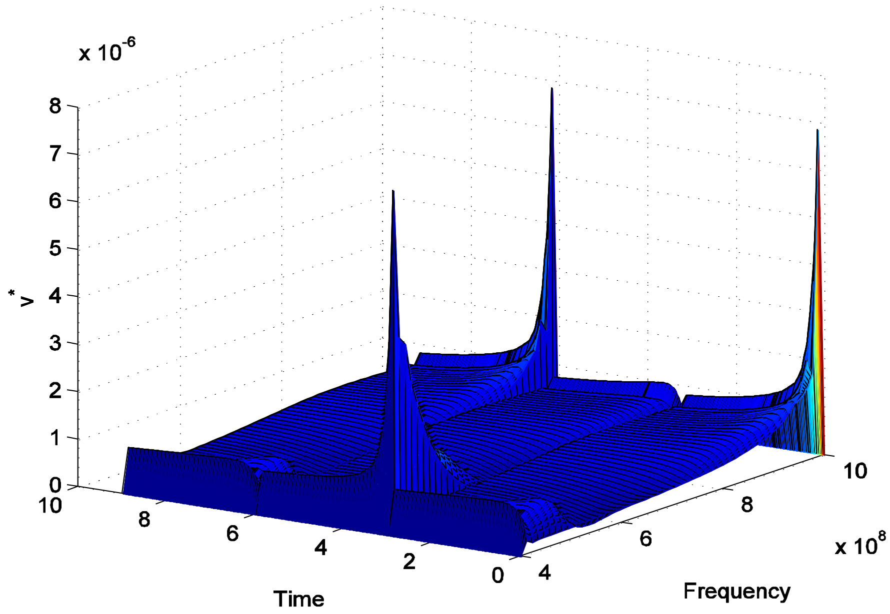


conditional density for amplitude of reflections

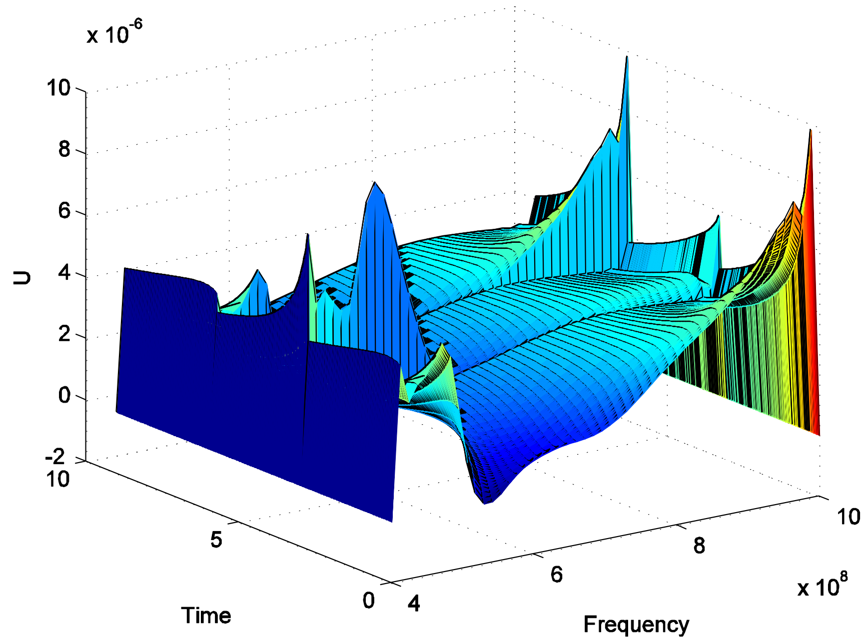
adaptive (feedback) control via changes in dielectric surfaces of target



Varying strategies: Beta-Gamma-Beta



conditional density for amplitude of reflections



adaptive (feedback) control via changes in dielectric surfaces of target

Motivation: distributional or generalized controls-extending static theory of [BGIT] where uncertainty in controls is embodied in probability measures on static control parameters such as dielectric permittivities and interrogating frequencies--rich literature on closure theorems--calculus of variations and optimal control --distinguished contributors such as Young (1937), McShane ('40,'67), Filippov('62), and Warga('62-'72)--some variational and control problems (and especially in two player differential games-see for example discussions Elliott('73) and the counter example of ***Berkovitz ('64)*** *) been known since years of L.C. Young that one must often introduce generalized or relaxed controls (also called sliding regimes or chattering controls) in order to obtain well posed optimization problems-- In anticipation two player dynamical games where both evader and interrogator have time dependent controllers, use generalized controls for evader (also introduces uncertainty in evader controls as well as uncertainty in interrogation frequencies via SDE dynamics)*

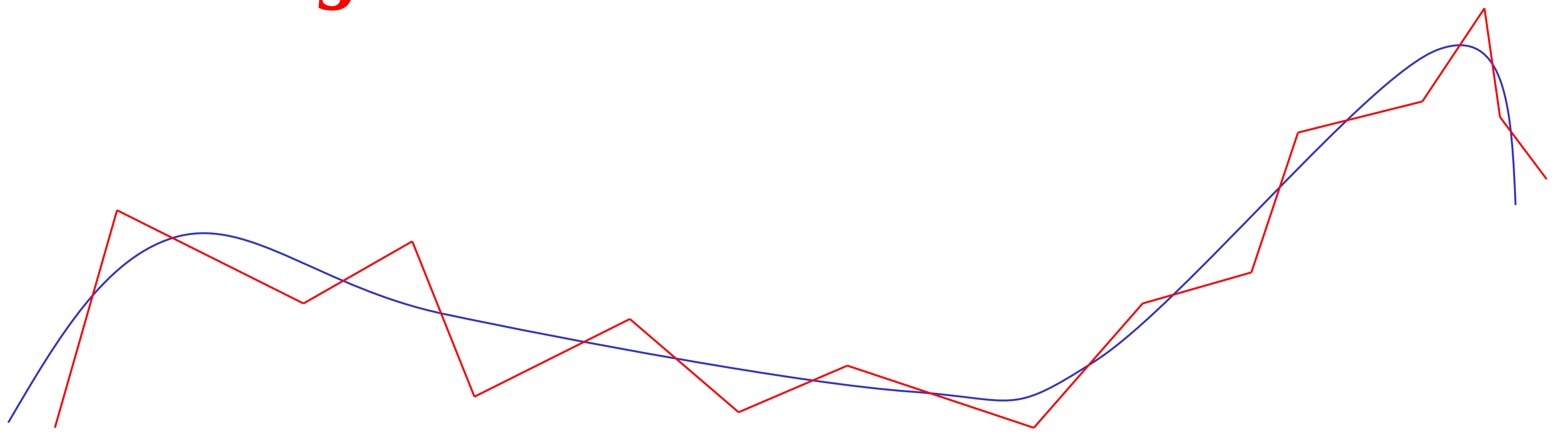
HTB and Shuhua Hu, A zero-sum electromagnetic evader-interrogator differential game with uncertainty, CRSC-TR11-04, N.C. State University, Raleigh, NC, February, 2011; *Applicable Analysis*, submitted.

Relaxed Controls
(sliding regimes, chattering controls)

L.C.Young (1937,1938)--generalized curves in calculus of variations,
E.J McShane (1940,1967)--relaxed controls in variational problems, control,
A.F.Filippov (1962)--sliding regimes in optimal control
J.Warga, (1962,1967,..)--relaxed curves and controls-approximation to "original" controls

Idea: Problems lack "closure" in ordinary function spaces (Re: Sobolev spaces and "weak" solutions, distributions of Schwartz for PDE)

Chattering controls:



In the limit, derivatives approach "limits of combinations of delta functions"

$$\frac{dx}{dt} = \lim \sum w_j f(t, x(t), u_j)$$

$$\frac{dx}{dt} = f(t, x(t), u(t)) \quad u(t) \in U$$

$$\frac{dx}{dt} = \sum w_j f(t, x(t), u_j) \qquad u_j \in U$$

$$\frac{dx}{dt} = \int_U f(t, x(t), u) d\mu(t, u)$$

$$= \mathcal{M}[f(t, x(t), \bullet); t]$$

Here $\mathcal{M}[\ \bullet\ ; t]$ *is an "averaging" operator (mean) over* U.

$\mathcal{M}[\ \bullet\ ;t]$ *is a time dependent probability measure*

$$\mathcal{M}[\ \bullet\ ;t] \leftrightarrow \mu(t,\ \bullet\ ) \in \mathcal{C}(U)^*$$

$$\mathcal{M}[\ \varphi(\bullet)\ ;t] = \int_U \varphi(u)\, d\mu(t,u) \ \text{ for}$$

$$\varphi \in \mathcal{C}(U),\ \mu(t,\ \bullet\ ) \in \mathcal{P}(U) \subset \mathcal{C}(U)^*$$

Recall : *Prohorov convergence* $\cong$

weak* *topology on* $\mathcal{C}(U)^*$

Control problem becomes:

Minimize $J(x(\mu), \mu)$

over $\mu(t, \bullet) \in \mathcal{P}(U) \subset \mathcal{C}(U)^*$

subject to

$$\frac{dx}{dt} = \int_U f(t, x(t), u)\, d\mu(t, u)$$

$$= \mathcal{M}[f(t, x(t), \bullet); t]$$

Convexifies and "generalizes" problem---obtain closure, and hence existence!

Corresponding trajectories are called "relaxed" trajectories or "generalized" trajectories (in which family of "ordinary" trajectories are embedded)

Existence (inf = min) of optimal controls in a wider class of functions—

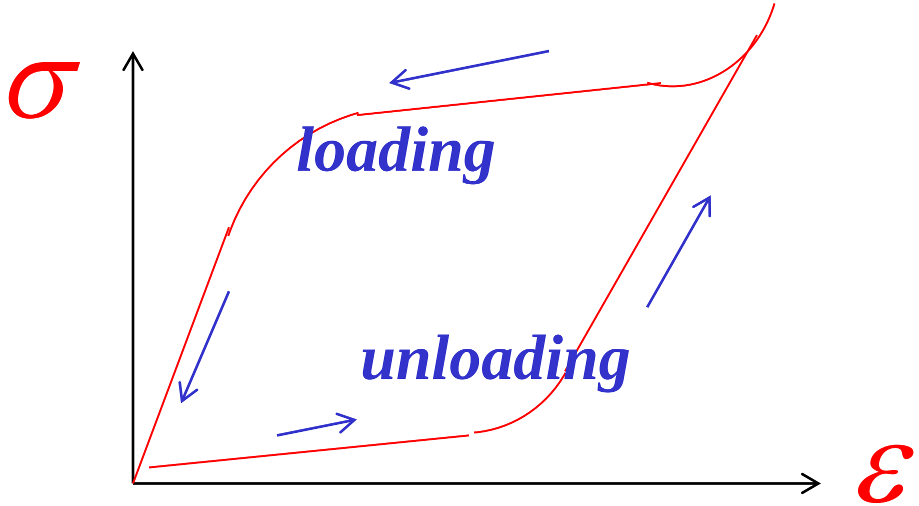
McShane: Under not too restrictive assumptions, inf actually attained by generalized control that happens to be ordinary!

Warga: Knowledge of minimizing generalized curve permits approximation by nearby ordinary curve!

PREISACH HYSTERESIS

1) ***Smart materials***
   SMA, piezoelectric, magnetostrictive
2) ***Viscoelastic materials***
   Rubber, polymers, living tissue
3) ***Electromagnetics***
   Polarization, conductivity

Example: Stress-strain in shape memory alloys, rubber, tissue



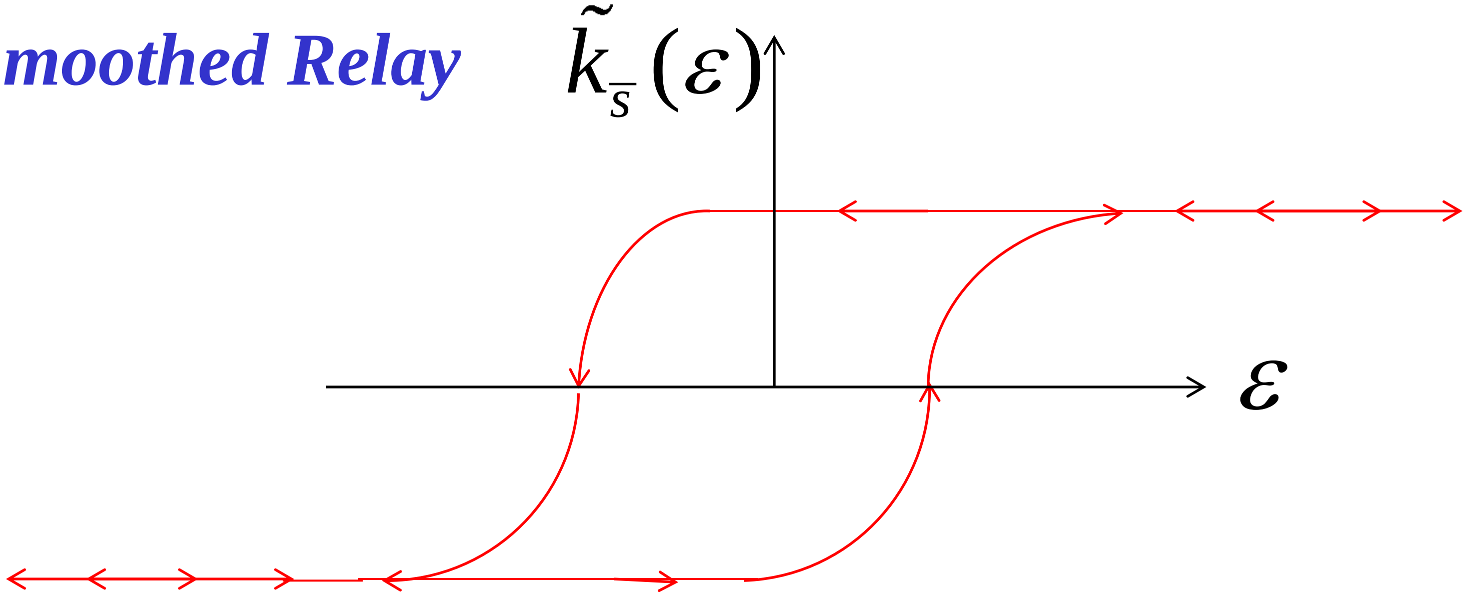
These ideas are the basis of SMA control efforts in:

H.T.Banks and A.J.Kurdila, Hysteretic control influence operators representing smart material actuators: Identification and approximation, *Proc. Conf. Decision and Control*, Kobe, Japan, Dec, 1996, p.3711-3716.

H.T.Banks, A.J.Kurdila and G.Webb, *Math. Prob. in Engr.*,3(1997),p.287-328.

H.T.Banks, A.J.Kurdila and G.Webb, *J. Intell. Mat. Sys.&Structures*,8(1997),p.536-550.

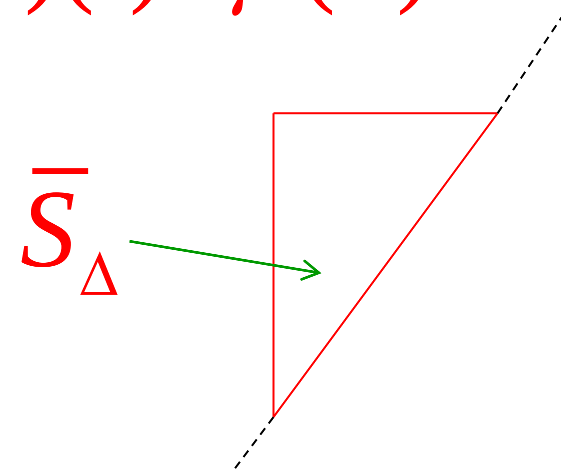
Smoothed Relay



for a continuum of switch points $\bar{s} = (s_1, s_2)$

then $$\sigma(\varepsilon)(t) = \int_{\bar{S}_\Delta} \tilde{k}_{\bar{s}}(\varepsilon)(t)\, d\mu(\bar{s})$$

for $\mu \in \mathscr{P}(\bar{S}_\Delta)$



$Estimation\ problem\ (for\ hysteretic\ material$

$properties)\ becomes:$

$Use\ MLE\ or\ OLS\ with\ data\ to\ optimize\ over$

$$\mu \in \mathcal{P}(\bar{S}_{\Delta}) \subset \mathcal{C}(U)^*$$

$subject\ to\ system\ dynamics\ with\ stress-strain$

$given\ by$ $$\sigma(\varepsilon)(t) = \int_{\bar{S}_{\Delta}} \tilde{k}_{\bar{s}}(\varepsilon)(t)\, d\mu(\bar{s})$$

Prohorov again!!!!